A general critique of inertial-electrostatic confinement fusion systems

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The suitability of various implementations of inertial-electrostatic confinement (IEC) systems for use as D–T, D–D, D–^{3}He, ^{3}He–^{3}He, p–^{11}B, and p–^{6}Li reactors has been examined, and several fundamental flaws in the concept have been discovered. Bremsstrahlung losses for all of these fuels have been calculated in a general fashion which applies not only to IEC systems but also to most other fusion schemes; these calculations indicate that bremsstrahlung losses will be prohibitively large for ^{3}He–^{3}He, p–^{11}B, and p–^{6}Li reactors and will be a considerable fraction of the fusion power for D–^{3}He and D–D reactors. Further calculations show that it does not appear possible for the dense central region of a reactor-grade IEC device to maintain significantly non-Maxwellian ion distributions or to keep two different ion species at significantly different temperatures, in contradiction with earlier claims made about such systems. Since the ions form a Maxwellian distribution with a mean energy not very much smaller than the electrostatic well depth, ions in the energetic tail of the distribution will be lost at rates greatly in excess of the fusion rate. Even by using one of the best electron confinement systems proposed for such devices, a polyhedral cusp magnetic field, and by making exceedingly optimistic assumptions about the performance of that confinement system, the electron losses from the machine prove to be intolerable for all fuels except perhaps DT. In order for IEC systems to be used as fusion reactors, it will be necessary to find methods to circumvent these problems. © *1995 American Institute of Physics.*

I. INTRODUCTION

Inertial-electrostatic confinement (IEC) involves the creation of deep electrostatic potential wells within a plasma in order to accelerate ions to energies sufficient for fusion reactions to occur and to keep the ions confined. It has been proposed to create and maintain these potential wells by a slight excess of electrons in a certain region of the plasma or by electrostatic grids. Typically such systems are arranged in a spherical geometry [leading to an alternate name sometimes used, Spherical Continuous Inertial Fusion (SCIF)], as illustrated in Figure 1 in Ref. 1. The ions converge in a radial manner toward the center of the plasma to form a very small, very dense core. Some of the earliest work on such systems was performed by Elmore, Tuck, and Watson,[2] Farnsworth,[3,4] Hirsch,[5,6] and Lavrent'ev.[7–10] Early research which was similar but used a toroidal geometry centered on the HIPAC (high-energy particle accelerator concept).[11–13] Yet another implementation of IEC has been described by Barnes, Turner, *et al.*;[14–16] in this incarnation, Penning traps or multipolar traps would be used to confine the particles.

Recently Bussard[1,17] has revived an idea originally suggested by Lavrent'ev[10] and proposed to surround an IEC electrostatic potential well with a polyhedral cusp magnetic field in order to improve electron confinement; this type of system has been analyzed by Krall, Rosenberg, and Wong[18–21] and is depicted in Figure 2 in Ref. 1.

Another recent suggestion by Bussard,[22] as well as Barnes and Turner,[23] is to use driven acoustic standing waves to increase the average density in the core of the device. Bussard refers to such a technique as the inertial-collisional compression (ICC) effect, and this paper will adopt his nomenclature in referring to this general method.

It has been suggested[1,19] that IEC can maintain non-Maxwellian particle distributions at fusion reactor parameters; it has been further suggested in Ref. 1 that two different species of fuel ions may be kept at significantly different energies. Both of these properties would confer the ability for the fusion device to exploit resonance peaks in fusion cross sections more fully than other systems can. Such a machine would then be highly suitable for use with advanced aneutronic fuels, thus greatly reducing radiation shielding requirements and the problems of structural material activation, as well as potentially allowing highly efficient direct electric conversion of some or all of the charged reaction products[24–27] (although, see the cautionary warnings about direct converter designs in Ref. 28). Coupled with the promise of high power densities and relatively simple engineering designs, these potential properties have made IEC systems very attractive.

The object of this paper is to examine various critical physics issues in as general a fashion as possible, so that the results will apply to a wide range of IEC systems and related variants. In particular, the potential problems which are analyzed include ion thermalization, ion losses, electron losses, electron thermalization, bremsstrahlung emission, and synchrotron radiation losses. As will be seen in the course of the analysis, radiation losses prohibit net energy production from all fuels except D–T, D–D, and D–^{3}He, while even extremely optimistic estimates indicate that severe electron losses will prevent successful operation with any fuel except possibly D–T. A serious problem with all fuels is that ions

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thermalize and escape over the top of the electrostatic well much more rapidly than they fuse.

Another potential problem with these concepts is the spreading of the core (degradation of focusing) due to the buildup of particle angular momentum acquired in collisions away from the exact center of the device. One initial analysis of this problem indicated that such harmful effects of collisions within the central region of the plasma (but not at the exact center) would be counteracted by beneficial "edge cooling" in the perpendicular directions out at the plasma boundary.[19] In order to place an optimistic upper bound on the performance of IEC devices, this paper will assume that the central focusing effect can indeed be maintained. (As will be seen, the resulting optimistic bound on IEC device performance is still too low to make these systems of interest for fusion reactor purposes.)

However, the calculations presented here will demonstrate that the particle velocity distributions in the central region of the device are essentially Maxwellian in the radial direction and that collisional effects in the edge are one to two orders of magnitude smaller than collisional effects occurring within the dense central region of the system. These results undermine the key tenets which led to the conclusions of Ref. 19. For these and other similar reasons, doubt has recently been cast on the ability of edge cooling or other effects to maintain reasonable convergence in the center.[29] If convergence is in fact lost, the performance of IEC devices will be far worse than is calculated in this paper.

The following conventions are adopted throughout the paper. If a species j is monoenergetic or otherwise non-Maxwellian with a mean energy of $\langle E_j \rangle$, then its temperature is defined as $T_j \equiv \frac{2}{3}\langle E_j \rangle$. All quantities are in cgs units, with temperatures in ergs, except where otherwise noted.

II. DESIGN-INDEPENDENT PHYSICS ISSUES

All of the issues examined in this section are effects that are essentially independent of the precise IEC device design; in particular, they are independent of the densities and density profiles which are used in the inertial-electrostatic fusion device (except for the very weak dependence on density contained in the Coulomb logarithm).

In performing these generalized calculations, the following assumptions have been made:

- For good convergence (i.e. systems in which the core radius r_c is much smaller than the radius R of the entire plasma system, $r_c \ll R$) the central region of an IEC device is dominant for effects such as fusion, bremsstrahlung, ion-electron energy transfer, and thermalization. Due to conservation of particles moving radially in the device, the particle density varies as approximately $n \approx n_c (r_c/r)^2$ (where n_c is the core density) outside of the core where radial velocity is approximately constant; fusion, bremsstrahlung, ion–electron energy transfer, and thermalization are all two-body effects, so they are all proportional to $n^2 \times \text{volume} \propto 1/r$, justifying this assumption. (The validity of this assumption will be demonstrated in a more detailed fashion in Sec. III.)
- In comparing fusion, bremsstrahlung, and collisional scattering rates, the density, spatial density profiles, and plasma volume do not matter, since all of the rates are proportional to $n^2 \times$volume [neglecting the weak density dependence of the Coulomb log].
- The dense central region may be considered approximately isotropic since particles are converging from and returning to all directions. (If it is not isotropic one must deal with problems such as Weibel and counter-streaming instabilities[30,31].)
- Spatial variations of temperature and energy may be neglected within the central region (assuming that the center of the potential well is fairly broad and flat, as stated in Refs. 1 and 18).
- There is no spatial variation in fuel stoichiometry (n_{i1}/n_{i2}, the ratio of the densities of the two ion species $i1$ and $i2$) in the region of appreciable density.
- Quasineutrality ($n_e = Z_{i1} n_{i1} + Z_{i2} n_{i2}$, in which n_e is the electron density, while Z_{i1} and Z_{i2} are the charges of the two ion species) holds in the region of significant density.
- The good convergence properties alleged for these devices[19] will be assumed to be valid for the purpose of the present calculations. Note, however, that there is good reason to believe that core convergence will be rapidly degraded by angular momentum buildup due to collisions.[29] Thus the present calculations represent an optimistic upper bound on the performance of IEC devices.

A. Fusion power density

The gross power per volume V produced by the fusion of two different ion species $i1$ and $i2$ is

$$\frac{P_{\text{fus}}}{V} = \langle \sigma v \rangle E_{\text{fus}} n_{i1} n_{i2}$$

$$= 1.602 \cdot 10^{-19} \langle \sigma v \rangle E_{\text{fus}} \frac{x}{(x+Z_2)^2} n_e^2 \, \frac{\text{W}}{\text{cm}^3}, \qquad (1)$$

where σ is the fusion cross section, v is the relative collision velocity, E_{fus} is the energy (in eV) released per reaction, and n_{i1} and n_{i2} are the densities of the two ion species. All quantities other than energy are in cgs units. The fusion power has also been rewritten in terms of the electron density n_e and the ratio of the ion densities, $x \equiv n_{i1}/n_{i2}$. It has been assumed here that the first ion species has a charge of one and the second ion species has charge Z_2, so the quasineutrality condition $n_e = n_{i1} + Z_2 n_{i2}$ was used in rewriting the fusion power.

Note that the fusion power is maximized for $x = Z_2$. Contrary to what one may initially think, the power is not maximum for $x = 1$, since it is the total charge (as represented by n_e, the electron density), not total number of ions, which is being held constant as the fuel mixture is changed. The total charge is limited in general by the structure and strength of the confining electric and magnetic fields. Expressing quantities explicitly in terms of a given electron density will also be convenient in the discussions of bremsstrahlung and electron particle losses which are to follow shortly.

If only a single ion species is reacting (e.g., D–D reactions), $n_{i1}n_{i2}$ in Eq. (1) should be replaced by $\frac{1}{2}n_i^2$ to avoid counting the same reactions twice. This change is equivalent to using the following substitution in the rewritten fusion power formula:

$$\frac{x}{(x+Z_2)^2} \rightarrow \frac{1}{2Z_i^2}. \tag{2}$$

For the purpose of comparisons with other characteristic times in the device, the characteristic fusion time of a test $i1$ ion with a member of the $i2$ ion species is readily defined (here $n_{i\text{ eff}}$ is the effective ion density seen by the test ion as it transits the system):

$$\tau_{\text{fus}} = \frac{1}{n_{i2\text{ eff}}\langle\sigma v\rangle}. \tag{3}$$

For the case of like-ion fusion, Eq. (3) should have an additional factor of 2 on the right-hand side to avoid double counting.

B. Energy equilibration between ion species

It is worthwhile to check whether one ion species can be maintained at a significantly lower energy than the other ion species. To do so, it will be assumed that the $i1$ species is more energetic than the $i2$ species and that the standard Spitzer-type expression for interspecies energy transfer may be applied to this problem. (As will be shown shortly, the individual ion species are essentially Maxwellian, and as demonstrated in Ref. 32, even large temperature differences between species do not result in large deviations from the usual interspecies energy transfer rates; thus the standard formulas may be employed here.)

Considering for the moment only the heating of the $i2$ species by the $i1$ species, the power density (in eV/s cm^3) transferred to the $i2$ ions will be:[33–35]

$$\begin{aligned}\frac{P_{i1-i2}}{V} &= \frac{3}{2}\, n_{i2}\, \frac{dT_{i2}}{dt} \\ &= 2.63\cdot 10^{-19}\, \frac{\sqrt{m_{i1}m_{i2}}Z_{i1}^2 Z_{i2}^2 n_{i1} n_{i2} \ln \Lambda_{i1-i2}}{(m_{i1}T_{i2}+m_{i2}T_{i1})^{3/2}} \\ &\quad \times (T_{i1}-T_{i2}),\end{aligned} \tag{4}$$

in which the temperatures are in eV.

It is now possible to consider two distinct cases. In the first case, T_{i2} is determined by balancing this heat transfer rate from $i1$ ions with the cooling effect due to the replacement of fused $i2$ ions with cold $i2$ ions. The second case is the situation in which the $i2$ ions are somehow actively refrigerated to ensure that they remain at very low energies.

Proceeding with the evaluation of the first case, the cooling rate of $i2$ ions due to the replacement of fused ions is

$$\frac{P_{\text{cool}}}{V} = \frac{3}{2}\, T_{i2} n_{i1} n_{i2} \langle\sigma v\rangle. \tag{5}$$

The equilibrium temperature of the $i2$ species is determined by setting the total amounts of heating and cooling equal to each other. Since both the heating and cooling expressions have the same dependence on the ion densities, integrating them over the spatial region of interest has no effect on the ratio between them. By defining the ion mass as a multiple of the proton mass m_p, $m_i \equiv \mu_i m_p$, and expressing the temperatures in eV, one arrives at an expression which is convenient for seeing the general range of permitted values for T_{i2}:

$$T_{i2} = T_{i1}\left[1 + \frac{7.40\cdot 10^6 \langle\sigma v\rangle (\mu_{i1}T_{i2}+\mu_{i2}T_{i1})^{3/2}}{\sqrt{\mu_{i1}\mu_{i2}}Z_{i1}^2 Z_{i2}^2 \ln \Lambda_{i1-i2}}\right]^{-1}. \tag{6}$$

For all of the fuels of interest (D–T, D–^{3}He, p–^{11}B, etc.) utilized under any reasonable circumstances (see, for example, the parameters in Tables I and II), one finds from Eq. (6) that the temperature of the $i2$ species is constrained to be very close to that of the $i1$ species:

$$0.95 T_{i1} \leqslant T_{i2} \leqslant T_{i1}. \tag{7}$$

Therefore, from this evaluation it does not appear possible to keep one ion species at a significantly lower temperature or energy than the other without providing additional means of cooling the $i2$ species.

Moving on to the second case in which a large temperature difference between the ion species is maintained by somehow actively refrigerating one species, it can be shown that the energy transfer rate required to sustain the nonequilibrium state would be prohibitively large. For this purpose assume that $T_{i1} \gg T_{i2}$, so that the collisions between the two ion species occur at a relative velocity $v \approx \sqrt{3T_{i1}/m_{i1}}$. Coulomb collisions will then transfer energy between the species at the rate calculated above. Dividing this energy transfer rate by the fusion power and putting T_{i1} and E_{fus} in eV, one obtains

$$\frac{P_{i1-i2}}{P_{\text{fus}}} = 1.20\cdot 10^{-13}\left(\frac{m_{i1}}{m_{i2}}\right)\frac{Z_{i1}^2 Z_{i2}^2 \ln \Lambda_{i1-i2}}{\sigma E_{\text{fus}} T_{i1}}. \tag{8}$$

For a numerical estimate it is illustrative to use the case of p–^{11}B reactions, for which it would be desirable to have high-energy protons ($i1$ species) and low-energy boron ions ($i2$ species). The peak of the fusion cross section, $\sigma \approx 8\cdot 10^{-25}$ cm^2, occurs for a proton energy of about 620 000 eV, or T_{i1} equal to two-thirds of that energy. Estimating the Coulomb logarithm as approximately 15, the power ratio is found to be

$$\frac{P_{i1-i2}}{P_{\text{fus}}} \approx 1.4. \tag{9}$$

From this calculation it may be seen that even if it were possible to operate the reactor with boron ions maintained at very low energies, the boron ions would siphon off more power from the energetic protons than would be produced from the nuclear reactions. (Similarly large numbers are found for reactors using other fuel ions as well.) In order to keep the system operating precisely at the resonance peak of the reaction cross section, it would then be necessary to extract the energy acquired by the boron ions and return this energy to the protons; otherwise, the mean energies of the two species would rapidly equilibrate, as in the first case. The task of finding a mechanism which can in this fashion continually recirculate a power substantially larger than the fusion power at very high efficiencies and extract the collisionally generated entropy from the system (without interfering with other elements of the fusion reactor's operation) appears daunting at best.

Thus both from the derivation of the natural energy equilibrium between ion species and from the calculated minimum recirculating power required to keep the system in the nonequilibrium energy state, it does not appear to be possible to maintain one ion species at a significantly higher energy than the other. For the remainder of this paper, it will therefore be assumed that the two ion species have the same mean energy (or temperature $T_i \equiv (2/3)\langle E_i \rangle$).

Even if both species can still be kept monoenergetic (but at approximately the same energy), the fact that $\langle \sigma v \rangle$ must be averaged over all collision angles then implies that it is impossible to exploit the resonance peaks of fusion cross sections (e.g. the sharp peaks in the p–^{11}B cross section) as fully as might be hoped.

C. Ion thermalization

The problem of ion thermalization and energy upscattering can be described in a straightforward manner. A test ion is injected into the well at the desired energy and begins to oscillate through the dense core, out toward the plasma edge, and back again. Collisions with other ions, all presumably starting at the same energy, will cause the test ion to diffuse in velocity space. The perpendicular velocity–space diffusion is less important than parallel diffusion, since most scattering occurs near the device center, and the well then returns the scattered particles to the center for another try. (Core spreading may still be a fatal problem,[29] but it will be ignored here to simplify the calculation and place an optimistic upper bound on the performance of IEC fusion reactors.) For this reason, the present analysis has focused on parallel velocity–space diffusion, or energy up- and downscattering.

Since the test ion only spends a fraction of its time in the core, it would be incorrect to compute the ion–ion collision time using the core density; rather, one must use some sort of effective density seen by the ion as it transits the entire system. Because this effective density is typically much smaller than the core density, the thermalization and upscattering times will be significantly lengthened.

Specifically, the ions will begin to evolve from their assumed initial monoenergetic distribution toward a Maxwellian distribution on a time scale characterized by the ion–ion collision time:[36]

$$\tau_{i1-i1} = \frac{3\sqrt{3}\sqrt{m_{i1}}T_{i1}^{3/2}}{8\pi Z_{i1}^4 e^4 n_{i1\ \mathrm{eff}} \ln \Lambda_{i1-i1}} = 1.0 \cdot 10^7 \frac{\sqrt{\mu_{i1}}T_{i1}^{3/2}}{Z_{i1}^4 n_{i1\ \mathrm{eff}} \ln \Lambda_{i1-i1}}, \tag{10}$$

where T_{i1} on the extreme right-hand side of the equation has been converted into eV.

The energy upscattering time required for an ion to escape over the top of the electrostatic well is related to the collision time and will be calculated in the next section.

The characteristic thermalization time for the test ion must be compared with the characteristic fusion time for that ion, in order to determine whether the ion is likely to fuse first or be thermalized first. Since the test ion spends only a small fraction of its time in the core, its fusion time should be lengthened by the same factor and for the same reasons as the collision time. When one takes the ratio of the fusion and thermalization times, which are both inversely proportional to density, the two factors cancel each other, so that

$$\frac{\tau_{i1-i1}}{\tau_{\mathrm{fus}}} = 1.0 \cdot 10^7 \frac{\sqrt{\mu_{i1}}T_{i1}^{3/2}\langle \sigma v \rangle}{Z_{i1}^4 \ln \Lambda_{i1-i1}} \frac{n_{i2}}{n_{i1}}. \tag{11}$$

Once again using the typical values of $\langle \sigma v \rangle \sim 10^{-16}-10^{-15}$ cm^3/s, $T_{i1} \sim 5 \cdot 10^4 - 6 \cdot 10^5$ eV, and $\ln \Lambda_{i1-i1} \sim 15-20$, it is found that

$$\frac{\tau_{i1-i1}}{\tau_{\mathrm{fus}}} \sim 10^{-3} - 10^{-2}. \tag{12}$$

One finds the result to be the same as in other fusion reactor designs, namely that an ion thermalizes about two to three orders of magnitude faster than it fuses. Of course the high-energy tail will require several collision times to fill in, but even so it is apparent that the ion distributions will be essentially Maxwellian. It is also worth noting that the distribution will be truncated at the well depth energy, since the energetic tail can escape from the confining potential well.

A recent suggestion for overcoming the thermalization problem is that collisions within the dense central region of the plasma may behave in a highly anisotropic one-dimensional fashion rather than in a three-dimensional isotropic fashion,[37] and that the relaxation of the desired monoenergetic radial velocity distributions may therefore happen on a much longer time scale. This behavior would require that the space-charge repulsion from a "hard core" of extremely low-energy ions (or possibly electrons) trapped at the very center of the device would reflect incoming particles of the same species before they entered the isotropic region of the core.

The probability of success for this method would appear to be exceedingly small. For it to work as desired, virtually all of the particles must bounce straight back from the hard core so that they only encounter other particles in head-on collisions. Even if a repulsive core is established, a more realistic outcome would be that a considerable number of particles converging toward the center would be deflected somewhat to the side by the core (or by their own angular momentum) and then "sideswiped" by other incoming particles, leading to collisional behavior that is approximately isotropic. Another difficulty is that there will be a high turnover rate (due to the fast collisional rates) of the very low-energy particles which constitute the hard core, as they exchange places with some of the higher-energy particles in the system. Thus most of the particles in the device will get to circulate into the isotropic core and undergo collisions there. Finally, even if the dense central region of an IEC system can be made to behave in a strongly anisotropic fashion, this behavior would be highly detrimental when instabilities are considered.[31]

Because the ion distributions are Maxwellian to a good approximation, all of the fusion cross sections must be Maxwellian averaged. One must, therefore, make do with the same $\langle \sigma v \rangle$ values as are used in other fusion devices, and resonance peaks in the cross sections cannot be utilized more efficiently than in other types of reactors. In fact, because the high-energy tail of the Maxwellian is truncated at the well depth, the average fusion reactivities will actually be somewhat lower than truly Maxwellian-averaged quantities.

D. Ion upscattering losses

If the initial ion energy is not much smaller than the potential well depth, say half of the well depth, then ions will not have to be scattered terribly far out into the tail to be lost. Ions can be lost either by completely escaping from the system or by climbing high enough in the well that the strong magnetic field near the plasma boundary deflects them into useless orbits. Both of these effects require that the ions be upscattered by a certain increment in energy. They can cover the comparatively small corresponding distance in velocity space in just a few ion–ion collision times. Thus not only will the ions thermalize far more rapidly than they will fuse, but they will also escape the well much more rapidly than they fuse (except for ions like ^{11}B which see a much deeper well).

These ion losses due to radial energy upscattering can now be specifically calculated. Consider the upscattering of a test $i1$-species ion with charge Z_{i1} by field ions of both species. The ion distribution function is initially monoenergetic at a mean energy of E_{i0} (for both ion species, since it was shown in Sec. II B that it is essentially impossible to decouple the energies of two ion species in a practical manner.) This initially monoenergetic distribution rapidly relaxes to a Maxwellian distribution with a temperature $T_i=(2/3)E_{i0}$. An $i1$ ion will be lost when its energy reaches the loss energy $E_{i1\ \mathrm{loss}}$, determined by the well depth:

$$E_{i1\ \mathrm{loss}}=Z_{i1}e\Phi_{\mathrm{well}}, \tag{13}$$

with e defined to be positive.

Expressions for the upscattering rate out of a purely electrostatic confinement system due simply to collisions within the same species (here the $i1$ species) have been derived.[38,39] From these references, the average time for a test $i1$ ion to be lost from the electrostatic potential well is

$$\begin{aligned}\tau_{i1\ \mathrm{loss}}&=\exp\left(+\frac{Z_{i1}e\Phi_{\mathrm{well}}}{T_i}\right)\\&\quad\times\left[1-\exp\left(-\frac{Z_{i1}e\Phi_{\mathrm{well}}}{T_i}\right)\right]\frac{\sqrt{m_{i1}}T_i^{3/2}}{4\sqrt{2\pi}Z_{i1}^4e^4n_{i1}\ln\Lambda_{ii}}\\&=\frac{\sqrt{2\pi}}{3^{3/2}}\exp\left(+\frac{Z_{i1}e\Phi_{\mathrm{well}}}{T_i}\right)\\&\quad\times\left[1-\exp\left(-\frac{Z_{i1}e\Phi_{\mathrm{well}}}{T_i}\right)\right]\tau_{i1-i1}.\end{aligned} \tag{14}$$

The effect of collisions with the $i2$ ion species may be incorporated into this calculation by noting Sivukhin's expression for the parallel velocity–space diffusion of a test $i1$ ion (denoted by subscript t) in the presence of monoenergetic but isotropic distributions of the $i1$ and $i2$ species:[40]

$$\begin{aligned}D_{\|}&=\frac{4\pi Z_{i1}^2e^4\ln\Lambda_{ii}}{3m_{i1}^2v_{i1t}^3}\sum_i Z_i^2 n_{i\ \mathrm{eff}}v_i^2\\&\propto\left[1+\left(\frac{Z_{i2}}{Z_{i1}}\right)^2\left(\frac{n_{i2}}{n_{i1}}\right)\left(\frac{m_{i1}}{m_{i2}}\right)\right]\frac{1}{\tau_{i1-i1}},\end{aligned} \tag{15}$$

in which it has been assumed that the two ion species have the same mean energy. The same factor for modifying the collisional time scales due to a second ion species would also be found from Sivukhin's diffusion coefficient for Maxwellian background particles.

Therefore, the time required for a test $i1$ ion to be upscattered out of the potential well due to collisions with both ion species is

$$\begin{aligned}\tau_{i1\ \mathrm{loss}}&=\frac{\sqrt{2\pi}}{3^{3/2}}\left[1+\left(\frac{Z_{i2}}{Z_{i1}}\right)^2\left(\frac{n_{i2}}{n_{i1}}\right)\left(\frac{m_{i1}}{m_{i2}}\right)\right]^{-1}\\&\quad\times\exp\left(+\frac{Z_{i1}e\Phi_{\mathrm{well}}}{T_i}\right)\left[1-\exp\left(-\frac{Z_{i1}e\Phi_{\mathrm{well}}}{T_i}\right)\right]\tau_{i1-i1}.\end{aligned} \tag{16}$$

The fraction of ions that fuse is just the ratio of the loss and fusion times:

$$\begin{aligned}\frac{\tau_{i1\ \mathrm{loss}}}{\tau_{\mathrm{fus}}}&=4.9\cdot10^6\left[1+\left(\frac{Z_{i2}}{Z_{i1}}\right)^2\left(\frac{n_{i2}}{n_{i1}}\right)\left(\frac{m_{i1}}{m_{i2}}\right)\right]^{-1}\\&\quad\times\exp\left(+\frac{Z_{i1}e\Phi_{\mathrm{well}}}{T_i}\right)\\&\quad\times\left[1-\exp\left(-\frac{Z_{i1}e\Phi_{\mathrm{well}}}{T_i}\right)\right]\frac{\sqrt{\mu_{i1}}T_i^{3/2}\langle\sigma v\rangle}{Z_{i1}^4\ln\Lambda_{i1-i1}}\frac{n_{i2}}{n_{i1}},\end{aligned} \tag{17}$$

in which T_i and $(e\Phi_{\mathrm{well}})$ are in eV.

Equation (17) will be evaluated in detail for various fusion fuels in Sec. IV, but one should note here that for $Z_ie\Phi_{\mathrm{well}}/T_i\sim2$ or 3, ions will be upscattered and lost from the potential well after only a few ion–ion collision times. In practical terms, it may not even be possible to inject ions that deeply into the well with any accuracy, and it would be still more difficult to inject them more deeply to try to reduce the upscattering losses. Even if ions could be injected more deeply, for the ions to have the same energy the well depth would have to be increased by a corresponding amount. This increased well depth would in turn increase the power loss due to electrons escaping the system, and as will be shown later, the electron power loss is already intolerably large without the deeper injection.

It should also be observed that although ions with very low Z will be rapidly upscattered out of the potential well, those ions which are lost will leave most of their energy behind as they climb out of the well. Therefore, the power loss due to escaping ions will be much less than it otherwise would be. (But note that the escaping ions will carry with them any perpendicular energy they have acquired if angular momentum buildup is indeed a problem.) However, even if the actual power loss caused by escaping ions is rather small, it will be excessively inconvenient (and costly in terms of pumping requirements and fresh ion injection) if the ions escape too rapidly, and this limitation may prevent IEC devices from being practical fusion reactors.

A more complete calculation of energy upscattering should also include cooling of the fast ions due to electron drag. As a rough estimate of the importance of electron drag relative to ion–ion upscattering, one may consider the ratio of the ion–electron collision time to the ion–ion collision time, as defined in Ref. 36. Considering only a single ion species for simplicity, it may be seen that

$$\frac{\tau_{\mathrm{ie}}}{\tau_{\mathrm{ii}}} \approx 21\sqrt{\mu_i} Z_i \left(\frac{T_e}{T_i}\right)^{3/2}. \tag{18}$$

For the fuels and temperatures characteristic of the proposed IEC systems ($T_e \sim T_i$, as will be shown shortly), the ion–electron collision time is typically at least an order of magnitude larger than the ion–ion collision time. Therefore, it appears doubtful that electron drag effects will substantially reduce the ion upscattering losses.

Another possible mechanism for reducing the upscattering losses is cooling of the ions by charge exchange with neutrals. Unfortunately, while charge exchange may prevent ions from escaping the system, the neutrals themselves would be free to escape, potentially carrying a sizeable amount of energy out of the confinement system. Considerable technical problems may also result from the pumping requirements necessary to collect the large quantities of escaping neutrals or ions and replace them with fresh ions. Of course, the presence of appreciable charge exchange effects would ultimately serve to degrade the monoenergetic radial ion velocity distributions even more quickly than has already been calculated. Because of all of these reasons, the introduction of neutrals into the problem is not a useful solution.

E. Bremsstrahlung radiation losses

Now the bremsstrahlung losses will be derived in such a way that the results will be applicable not only to IEC systems but also to a wide variety of other reactor designs (basically any plasma system in which the regions of appreciable density are approximately isotropic, quasineutral, and optically thin to bremsstrahlung).

Maxon[41] gives the nonrelativistic and extreme relativistic limits for electron–ion and electron–electron bremsstrahlung and interpolates between these two limits to obtain approximate radiation rates in the intermediate regime. His results may be used as a guideline for the necessary corrections for moderately relativistic ($T_e \sim 100$ keV) electrons (consult McNally[42] for a commentary on the empirical expression for bremsstrahlung in this regime). Adding together the expressions for ion–electron and electron–electron bremsstrahlung and defining

$$Z_{\mathrm{eff}} \equiv \frac{\Sigma_i Z_i^2 n_i}{n_e} = \frac{x+Z_2^2}{x+Z_2} \tag{19}$$

produces the result that the total bremsstrahlung power per volume is

$$\frac{P_{\mathrm{brem}}}{V} = 1.69 \cdot 10^{-32} n_e^2 \sqrt{T_e} \left\{ Z_{\mathrm{eff}} \left[1 + 0.7936 \frac{T_e}{m_e c^2} + 1.874 \left(\frac{T_e}{m_e c^2} \right)^2 \right] + \frac{3}{\sqrt{2}} \frac{T_e}{m_e c^2} \right\} \frac{\mathrm{W}}{\mathrm{cm}^3}, \tag{20}$$

where both T_e and the electron rest energy $m_e c^2$ are in eV.

Once again using the usual expression for energy transfer between plasma species,[33–35] the heating of electrons by ions may be described by

$$\frac{P_{\mathrm{ie}}}{V} = 7.61 \cdot 10^{-28} n_e \sum_i \frac{Z_i^2 n_i \ln \Lambda_{ei}}{\mu_i T_e^{3/2}} \times \left(1 + \frac{m_e}{m_i} \frac{T_i}{T_e} \right)^{-3/2} (T_i - T_e) \frac{\mathrm{W}}{\mathrm{cm}^3}, \tag{21}$$

in which the ion masses have been expressed in multiples of the proton mass, $m_i = \mu_i m_p$, temperature is in eV, and density is in particles/cm^3.

As presented in Ref. 32, there is a correction factor to this classical Spitzer expression for ion–electron heating; the correction is caused by ion-induced partial depletion of the electrons with velocities smaller than the ion thermal velocity, with the net result that

$$\left(\frac{P_{\mathrm{ie}}}{V}\right)_{\mathrm{actual}} \approx \left(\frac{P_{ie}}{V}\right)_{\mathrm{Spitzer}} \left(1 + \frac{m_e}{m_i} \frac{T_i}{T_e} \right)^{3/2} \times \exp\left[- \left(3.5 \sum_i \frac{Z_i^2 n_i}{n_e} \frac{m_e}{m_i} \frac{T_i}{T_e} \right)^{2/3} \right]. \tag{22}$$

Also from Ref. 32, this form of the correction factor yields very good results for

$$\sum_i \frac{Z_i^2 n_i}{\mu_i n_e} \frac{T_i}{T_e} \leqslant 50. \tag{23}$$

For larger temperature ratios, Eq. (22) begins to underestimate the actual ion–electron heat transfer. Therefore, it will serve well as an optimistic bound on the electron heating and radiation loss problem.

The Coulomb logarithm also has a temperature dependence which should be taken into account. According to Ref. 35, the appropriate form of the log in the event of ion-electron collisions (for $T_i m_e / m_i < 10 Z_i^2$ eV$< T_e$) is

$$\ln \Lambda_{\mathrm{ei}} \approx 24 - \ln\left(\frac{\sqrt{n_e}}{T_e} \right), \tag{24}$$

in which n_e is in cm^{-3} as usual and T_e is in eV.

Furthermore, Dawson[43] notes that for relativistic electrons the ion–electron heating must be modified by a factor of $(1 + 0.3 T_e / m_e c^2)$. After incorporating all of these corrections, the heat transfer rate becomes

$$\frac{P_{\text{ie}}}{V}=7.61\cdot10^{-28}n_e\left(1+\frac{0.3T_e}{m_ec^2}\right)\sum_i\frac{Z_i^2n_i}{\mu_iT_e^{3/2}}\times\left[24-\ln\left(\frac{\sqrt{n_e}}{T_e}\right)\right]\exp\left[-\left(3.5\sum_i\frac{Z_i^2n_i}{n_e}\frac{m_e}{m_i}\frac{T_i}{T_e}\right)^{2/3}\right]\times(T_i-T_e)\ \frac{\text{W}}{\text{cm}^3} \tag{25}$$

for temperatures and the electron rest energy in eV.

The equilibrium electron temperature is found by equating the power transferred to the electrons by ion–electron heating with the power lost by the electrons due to bremsstrahlung, synchrotron radiation, ion–electron cooling in the edge of the device, loss of electrons from the system, and other effects. The maximum possible bremsstrahlung rate may be obtained by neglecting all loss mechanisms except bremsstrahlung, thus producing the highest possible equilibrium electron temperature (barring additional heat sources to the electrons other than simply Coulomb friction with the ions). In this approximation $P_{\text{ie}}=P_{\text{brem}}$. (In Sec. III both synchrotron radiation and edge ion–electron heat transfer will be shown to be negligibly small compared with the bremsstrahlung and heating effects considered here, so this approximation should come close to the actual answer.) Since the ion–electron heating and the bremsstrahlung cooling both have the same dependence on the densities (with the exception of the Coulomb logarithm, which slowly varies from about 15 to approximately 20 over the range of the system), integrating over the system volume has essentially no effect. As a result, the equilibrium electron temperature (in eV) can be determined from the general equation:

$$44\,940\left(1+\frac{0.3T_e}{m_ec^2}\right)\sum_i\frac{Z_i^2n_i}{\mu_in_e}\left[24-\ln\left(\frac{\sqrt{n_e}}{T_e}\right)\right]\times\exp\left[-\left(3.5\sum_i\frac{Z_i^2n_i}{n_e}\frac{m_e}{m_i}\frac{T_i}{T_e}\right)^{2/3}\right](T_i-T_e)=T_e^2\left\{Z_{\text{eff}}\left[1+0.7936\frac{T_e}{m_ec^2}+1.874\left(\frac{T_e}{m_ec^2}\right)^2\right]+\frac{3}{\sqrt{2}}\frac{T_e}{m_ec^2}\right\}. \tag{26}$$

After finding the equilibrium electron temperature, its value (still in eV) may be used to calculate the fraction of the gross fusion power output which is radiated away by bremsstrahlung:

$$\frac{P_{\text{brem}}}{P_{\text{fus}}}=1.06\cdot10^{-13}\frac{(x+Z_2)^2}{x}\frac{\sqrt{T_e}\{Z_{\text{eff}}[1+0.7936(T_e/m_ec^2)+1.874(T_e/m_ec^2)^2]+(3/\sqrt{2})(T_e/m_ec^2)\}}{\langle\sigma v\rangle E_{\text{fus}}}. \tag{27}$$

Equations (26) and (27) for the electron temperature and the ratio of the bremsstrahlung loss power to the fusion power will be evaluated for specific fuels in Sec. IV. However, as a brief look ahead, one would expect that since the ion distributions are essentially Maxwellian and the ion–electron heat transfer rate is not greatly modified from its classical Spitzer value, one will obtain the results already familiar for other fusion reactor types. In particular, while deuteron-based fuels can theoretically produce net power despite bremsstrahlung losses, more advanced fuels, such as p–^{11}B, should be unable to do so.[42]

As stated earlier, these values for the electron temperature and bremsstrahlung have been calculated assuming that bremsstrahlung is the dominant mechanism for cooling of the electrons. This assumption is justified because most of the other possible cooling effects (such as electron cusp losses) would cause an even greater net power loss while cooling the electrons and reducing the bremsstrahlung power loss. Although cooling of hot electrons by cold ions in the edge would be a beneficial effect which would not cause further power losses, the very low densities in the edge region stipulate that the total electron cooling there will be much less than the total electron heating in the center of the device, as will be explicitly shown in the next section.

III. DESIGN-DEPENDENT PHYSICS ISSUES

Included in this section are effects that depend on the specific density, density profiles, and confinement system (e.g., magnetic cusp, grids, etc.) which are employed. The first section will outline the specific assumptions made about the spatial density and energy profiles of the devices, and subsequent sections will use these profiles to calculate the magnitude of the various effects.

A. Spatial profiles

1. Devices with convergence-limited core densities

In the simplest IEC concepts, the core density is determined solely by the convergence of the spherical flow in the potential well. In the following calculations, it will be assumed that the device employs a single potential well. The theoretical analysis may be simplified by dividing the interior of the machine into three regions: the core ($0<r<r_c$), the mantle ($r_c<r<r_e$), and the edge ($r_e<r<R$). Typically $R\approx100r_c$ and $r_e\sim50-80r_c$. The following approximate forms for the particle densities and energies are assumed.

Both the electron and ion densities are constant in the core, then because of conservation of particles in the nearly flat part of the potential well, they drop off like $1/r^2$ in the mantle, and they finally reach a constant value in the edge:

$$n_{e,i}=\begin{cases}(n_c)_{e,i} & 0<r<r_c,\\ (n_c)_{e,i}(r_c/r)^2 & r_c<r<r_e,\\ (n_c)_{e,i}(r_c/r_e)^2 & r_e<r<R.\end{cases} \tag{28}$$

This is a useful approximation of the more accurate density profile given in Figure 3 in Ref. 18.

Electrons in the core and mantle are heated enough by Coulomb friction with the energetic ions that they will tend to form a Maxwellian distribution with a temperature of T_{e0}. As the electrons travel from the center to the edge of the well, they acquire additional energy corresponding to the well depth, so that the electron energy distribution is given by

$$\langle E_e\rangle=\begin{cases}\frac{3}{2}T_{e0} & 0<r<r_e,\\ \frac{3}{2}T_{e0}+e\Phi_{\text{well}}f_{\text{well}}(r) & r_e<r<R,\end{cases} \tag{29}$$

for which the well shape function $f_{\text{well}}(r)$ is some rapidly increasing function of r such that $f_{\text{well}}(r_e)=0$ and $f_{\text{well}}(R)=1$.

Similarly, the ion energies have the following spatial variation:

$$\langle E_i\rangle=\begin{cases}\dfrac{3}{2}\,T_{i0} & 0<r<r_e\\ \dfrac{3}{2}\,T_{i0}-e\Phi_{\text{well}}Z_i\,f_{\text{well}}(r) & r_e<r.\end{cases} \tag{30}$$

(Figure 3 in Ref. 18 presents a graph of the typical potential well shape.)

To a first approximation bremsstrahlung, fusion, and ion–electron heating in the edge may be neglected because of the low densities and ion energies there. Using the fact that $r_e \gg r_c$, the following useful integral over the core and mantle regions is found:

$$\int_0^{r_e} n^2 4\pi r^2\,dr \approx \frac{16}{3}\,\pi n_c^2 r_c^3. \tag{31}$$

2. Devices with enhanced core densities

Another class of IEC schemes is centered around increasing the core density beyond the convergence-limited value. References 22 and 23 propose to achieve this increase using acoustic standing waves; this phenomenon has been called the inertial-collisional compression (ICC) effect. If the only role of the standing waves is to increase the core density or change the density profile, there will be no significant impact on the problems which have already been analyzed. The primary usefulness of the ICC effect lies in its being able to affect the total fusion power and hence perhaps the relative severity of electron cusp losses.

If the root-mean-squared (RMS) core density of an ICC device is enhanced beyond that of a non-ICC device by a factor of ξ, then the density profile may be approximated as

$$n_{e,i}=\begin{cases}(n_c)_{e,i} & 0<r<r_c,\\ (n_c)_{e,i}(r_c/r)^2/\xi & r_c<r<r_e,\\ (n_c)_{e,i}(r_c/r_e)^2/\xi & r_e<r<R.\end{cases} \tag{32}$$

For simplicity the energy profiles will be considered to remain approximately like those in purely convergence-limited machines.

If the core density is significantly enhanced ($\xi \gg 1$) via the ICC effect or other mechanisms, then essentially only the core will contribute to processes like fusion and bremsstrahlung; the rate of these processes in the mantle will be negligible by comparison. The net result of this fact is that

$$\int_0^{r_e} n^2 4\pi r^2\,dr \approx \frac{4}{3}\,\pi n_c^2 r_c^3. \tag{33}$$

B. Relative importance of edge and central plasma regions

One of the key assumptions on which this entire analysis is based is that only the dense central region of the plasma contributes significantly to processes such as bremsstrahlung and ion–electron heat transfer; the edge region is assumed to make a negligible contribution to these processes. This assumption can now be justified.

Two-body effects such as bremsstrahlung and ion–electron heat transfer can be expressed as total powers and are proportional to the density squared times the volume of the region concerned. For a given process, the ratio of the powers of that process in the edge and central (combined core and mantle) regions is

$$\frac{P_{\text{edge}}}{P_{\text{core+mantle}}} \sim \frac{\int_{r_e}^{R} n^2 4\pi r^2\,dr}{\int_0^{r_e} n^2 4\pi r^2\,dr}. \tag{34}$$

For a device with convergence-limited core density, Eqs. (28) and (31) may be used to express the ratio of integrals from Eq. (34) as

$$\frac{\int_{r_e}^{R} n^2 4\pi r^2\,dr}{\int_0^{r_e} n^2 4\pi r^2\,dr}=\frac{1}{4}\left(\frac{r_c}{r_e}\right)\left[\left(\frac{R}{r_e}\right)^3-1\right]. \tag{35}$$

For typical values of $r_e=50r_c$ and $R=2r_e$, the ratio of edge effects to central region effects is

$$\frac{\int_{r_e}^{R} n^2 4\pi r^2\,dr}{\int_0^{r_e} n^2 4\pi r^2\,dr}=0.035. \tag{36}$$

The general value for this ratio is also supported by the more accurate density profiles of Figure 3 in Ref. 18.

For devices which have better core convergence or which employ the ICC effect, the ratio will be even smaller.

Of course, in computing the ratio of edge effects to central region effects for particular quantities like bremsstrahlung power or power transferred between ions and electrons, the exact answer will involve other numerical factors to account for parameters such as particle temperatures in the edge versus in the core, Coulomb logarithms in the two different regions, etc. Precise evaluation of these factors requires detailed spatial profiles of the electron temperature and ion temperature in the edge region, but in general the net result of the factors will be to change the result above by at most a factor of 2 or 3. In conclusion, one finds that $P_{\text{edge}}/P_{\text{core+mantle}} \sim 10^{-2}-10^{-1}$, for any power P such as the bremsstrahlung power, ion–electron power transfer, etc. It is clearly evident that for the purposes of the calculations presented in this paper, the edge region can be neglected in comparison with the central region of the plasma for both non-ICC and ICC designs.

This finding, together with the observation that the ion velocity distributions will be essentially thermal in the radial direction, strongly suggests that the analysis of angular momentum buildup presented in Ref. 19 should be closely reexamined.

Even if it were possible to make the value of $\int n^2\, d^3\mathbf{x}$ for the edge comparable to that of the central region, it would not be desirable to do so. bremsstrahlung, synchrotron radiation, and electron particle losses from the edge region (where the electron energy is very high) would become far worse than they otherwise would be.

C. Total fusion power

Using the integral in Eq. (31) for a machine with convergence-limited core densities, the total fusion power is found to be

$$P_{\text{fus}}=2.68\cdot 10^{-18}\langle\sigma v\rangle E_{\text{fus}}\frac{x}{(x+Z_2)^2}n_{ce}^2 r_c^3 \text{ W}, \tag{37}$$

where $x\equiv n_{i1}/n_{i2}$, $Z_{i1}=1$, $Z_{i2}=Z_2$, and E_{fus} is in eV. The fusion power is maximized for $x=Z_2$. Recall that if only one ion species is present, one should make the substitution (2) in the fusion power.

For a device employing the ICC effect, Eq. (37) has an additional factor of 1/4 on the right-hand side.

D. Electron cusp losses

An especially serious power loss mechanism is the loss of energetic electrons through the cusps of the confining magnetic field. At conditions of interest for a fusion reactor, an optimistic assumption about the effective radius r_{H} of each point cusp "hole" through which electrons escape is that it is of the order of the electron gyroradius ρ_e, so that $r_{\text{H}}=k_{\text{H}}\rho_e$, where it has been estimated that $1\leqslant k_{\text{H}}<5$.[44,45] The electron gyroradius, in turn, is given by the formula[35]

$$\rho_e=2.38\frac{\sqrt{2E_e}}{B}\text{ cm}, \tag{38}$$

in which E_e is the electron energy in eV and B is the magnetic field in Gauss. Note that a factor of $\sqrt{2}$ has been introduced into the usual formula because at the outer surface of the plasma the electrons are in directed motion.[1]

Now an expression for the characteristic electron loss time may be derived. One begins by noting that the fraction of electrons lost during each pass through the system will just be the total area of the cusp holes divided by the surface area of the machine. If this loss fraction is made small enough to be practical, then G_e, the average number of transits an electron makes through the system before being lost, is well approximated as just the inverse of the loss fraction. Specifically, for a magnetic field configuration which has N such point cusps (and with E_e still in eV):

$$G_e=\frac{4\pi R^2}{N\pi r_{\text{H}}^2}=0.353\frac{R^2B^2}{Nk_{\text{H}}^2E_e}. \tag{39}$$

The loss time may be expressed as $\tau_{e\text{ loss cusp}}=G_e\tau_{\text{tr}}$, where τ_{tr} is the time required for a single transit through the system. This transit time is in turn $\tau_{\text{tr}}=2R/v_{\text{av}}$, where v_{av} is the average electron velocity, or

$$v_{\text{av}}=A\sqrt{\frac{T_{e0}}{m_e}}, \tag{40}$$

with A some number of order unity to account for the faster electron speed near the edge of the well.

Putting the above equations together and using $E_e\approx e\Phi_{\text{well}}$, one obtains the electron loss time,

$$\tau_{e\text{ loss cusp}}\approx 1.68\cdot 10^{-8}\frac{R^3B^2}{Nk_{\text{H}}^2Ae\Phi_{\text{well}}\sqrt{T_{e0}}}\text{ s}, \tag{41}$$

where the temperature and energy are in eV and everything else is in cgs units.

One may now derive the power loss due to electrons leaking through the magnetic cusps by using $\tau_{e\text{ loss cusp}}$, as given in Eq. (41). If the total electron population in the machine is N_e and the energy per lost electron is approximately $e\Phi_{\text{well}}$ (in ergs), then the power loss due to escaping electrons is

$$P_{e\text{ loss cusp}}\approx\frac{N_e e\Phi_{\text{well}}}{\tau_{e\text{ loss cusp}}}. \tag{42}$$

Using the density profile of Eq. (28), the total electron population is found to be

$$N_e=\frac{4}{3}\pi n_{ce}r_c^2r_e\left[2\left(1-\frac{r_c}{r_e}\right)+\left(\frac{R}{r_e}\right)^3\right]. \tag{43}$$

For typical length ratios within the machine, $r_c/r_e\ll 1$, so it may be neglected compared with the other two terms within the brackets.

Putting these equations together and expressing $e\Phi_{\text{well}}$ and T_{e0} in eV, one obtains

$$P_{e\text{ loss cusp}}\approx 3.98\cdot 10^{-11}A\left(\frac{r_e}{R}\right)\left(\frac{r_c}{R}\right)^2\left[2+\left(\frac{R}{r_e}\right)^3\right]\times Nk_{\text{H}}^2\frac{(e\Phi_{\text{well}})^2\sqrt{T_{e0}}n_{ce}}{B^2}\text{ W}. \tag{44}$$

Now one makes the assumption that the edge of the plasma has $\beta=1$,[10,46] or $B^2=8\pi n_{\text{edge }e}e\Phi_{\text{well}}$, for $e\Phi_{\text{well}}$ in ergs. With the well depth energy and temperature in eV and using $n_{\text{edge }e}=n_{ce}(r_c/r_e)^2$, the ratio of cusp loss power to fusion power can be simplified somewhat:

$$P_{e\text{ loss cusp}}\approx 0.988A\left[1+2\left(\frac{r_e}{R}\right)^3\right]Nk_{\text{H}}^2e\Phi_{\text{well}}\sqrt{T_{e0}}\text{ W}. \tag{45}$$

The ratio of the electron loss power to the fusion power of an IEC device with convergence-limited core densities is

$$\frac{P_{e\text{ loss cusp}}}{P_{\text{fus}}} \approx 3.69\cdot 10^{17} A\left[1+2\left(\frac{r_e}{R}\right)^3\right] \times \frac{(x+Z_2)^2}{x}\frac{Nk_{\text{H}}^2 e\Phi_{\text{well}}\sqrt{T_{e0}}}{\langle\sigma v\rangle E_{\text{fus}} n_{ce}^2 r_c^3}, \quad (46)$$

where temperatures and energies are still expressed in eV.

This electron power loss fraction is minimized for $x=Z_2$.

Several factors in the expression for cusp losses are determined by the shape and depth of the electrostatic well. Using typical values of $A\approx 1.5$, $r_e/R\approx 0.5$, and $T_{e0} \approx e\Phi_{\text{well}}/3$, Eq. (46) reduces to

$$\frac{P_{e\text{ loss cusp}}}{P_{\text{fus}}} \approx 4\cdot 10^{17}\frac{(x+Z_2)^2}{x}\frac{Nk_{\text{H}}^2(e\Phi_{\text{well}})^{3/2}}{\langle\sigma v\rangle E_{\text{fus}} n_{ce}^2 r_c^3}. \quad (47)$$

It must be emphasized that this expression for cusp losses is inherently optimistic in a number of ways. The Appendix presents a different derivation of cusp losses which produces a result approximately four times larger than the one here; while Eq. (46) for the cusp power losses is dependent on the exact potential well profile, density profile, and relative temperatures of the different particle species, the result of Eq. (A4) in the Appendix is not.

The cusp holes were assumed to have a radius of only twice the electron gyroradius ρ_e, when, in fact, they may be several times larger (according to work by Grad[44] and Grossman[45] the hole radius could be as large as five times the electron gyroradius) or even scale like the geometric mean of the electron and ion gyroradii, $\sqrt{\rho_e\rho_i}$, when ion effects are included.[47]

It was also optimistically assumed that very high core densities could be achieved and that Ohmic power losses in the field coils could be neglected. Furthermore, all electron losses other than through the N point cusps (e.g. via any line cusps that may exist, across magnetic field lines, etc.) were optimistically ignored.

One might be tempted to use electrostatic fields at the cusps in order to reduce the number of escaping electrons or the amount of energy which they carry away. Unfortunately, such techniques would also increase the ion losses or the energy carried away by each escaping ion, and so they are of little interest.

A better method to reduce the electron power losses is to direct–convert the energy of escaping electrons into electricity. In particular, the addition of a transverse magnetic field outside each cusp would result in $\mathbf{v}\times\mathbf{B}$ forces which could separate outgoing escaping electrons from incoming fresh electrons. Then the outgoing electrons could be efficiently directed around the electron guns, so that they would hit direct–converter grids and hopefully return most of their energy to the system. One problem with this approach, though, is that the escaping electrons will have a large thermal spread, and this variation of energies will limit the efficiency of the direct converters.

Even with all of these very optimistic assumptions, the electron losses prove to be sizeable for D–T and intolerable for all other fuels, as will be shown in Sec. IV. Under actual operating conditions, the cusp losses will almost certainly be significantly higher than calculated here, and even D–T would not be able to attain a positive power balance.

E. Electron grid losses

Now consider the power losses that are caused by confining the particles with an electrostatic grid instead of a magnetic cusp system. The ion losses on the grid can be minimized by making the grid bias large and positive, so that ions hitting the grid will possess essentially zero energy; one must then calculate only the electron grid losses. Assuming that the grid has radius r_{grid} and transparency η_e to electrons passing through it, and choosing the electron energy, velocity, and density to be evaluated at the grid, one then obtains

$$P_{e\text{ loss grid}} = (1-\eta_e)4\pi r_{\text{grid}}^2\, n_{\text{grid}\,e}\, v_{e\text{ grid}}\, E_{e\text{ grid}}. \quad (48)$$

Noting that $v_{e\text{ grid}} = \sqrt{2E_{e\text{ grid}}/m_e}$ and expressing the power in watts, energy in eV, and everything else in cgs units, one finds that

$$P_{e\text{ loss grid}} = 1.19\cdot 10^{-10}(1-\eta_e) r_{\text{grid}}^2\, n_{e\text{ grid}}\, E_{e\text{ grid}}^{3/2}\ \text{W}. \quad (49)$$

It is instructive to compare this expression for the grid losses with the earlier expression for total fusion power:

$$\frac{P_{e\text{ loss grid}}}{P_{\text{fus}}} = 4.45\cdot 10^7\frac{(x+Z_2)^2}{x}(1-\eta_e) \times\left(\frac{r_{\text{grid}}}{r_c}\right)^2\left(\frac{n_{e\text{ grid}}}{n_{ce}}\right)\frac{E_{e\text{ grid}}^{3/2}}{\langle\sigma v\rangle E_{\text{fus}} n_{ce} r_c}. \quad (50)$$

Taking $r_{\text{grid}}=R$, $n_{e\text{ grid}} = n_{e\text{ edge}} = n_{ce}(r_c/r_e)^2$, $E_{e\text{ grid}} = e\Phi_{\text{well}}$, and $x=Z_2$, this expression becomes

$$\frac{P_{e\text{ loss grid}}}{P_{\text{fus}}} = 1.78\cdot 10^8 Z_2(1-\eta_e)\left(\frac{R}{r_e}\right)^2\frac{(e\Phi_{\text{well}})^{3/2}}{\langle\sigma v\rangle E_{\text{fus}} n_{ce} r_c}, \quad (51)$$

where the energies are still in eV.

This ratio indicates that at typical reactor parameters the grid losses are several orders of magnitude greater than the fusion power. As an illustration of the optimum performance that can be expected, choosing $Z_2=1$, $\eta_e=0.99$, $R/r_e=2$, $e\Phi_{\text{well}}=60\,000$ eV, $\langle\sigma v\rangle=10^{-15}$ cm^3/s, $E_{\text{fus}}=2\cdot 10^7$ eV, $n_{ce}=10^{18}$ cm^{-3}, and $r_c=2$ cm, one discovers that

$$\frac{P_{e\text{ loss grid}}}{P_{\text{fus}}} \sim 3000. \quad (52)$$

Not only are the electron losses tremendously greater than the fusion power, but one also has the inherent problem of cooling the grids.

Although one might contemplate passing a current through the grid wires to create a magnetic field around them and reduce the number of particles striking the grid, this idea does not appear to be advisable. Magnetic fields strong enough to deflect particles from the grid wires would also interfere with the desired purely radial motion of the particles, thereby significantly reducing the degree of core convergence in the IEC system.

F. Ion grid losses

One could attempt to reduce the power losses by putting a large negative bias on the grids; then it will be the ions and not the electrons which would constitute most of the power loss upon impact with the grids. By analogy with the electron calculation, it is straightforward to derive the ion losses caused by an electrostatic grid in the system. If the grid ion transparency is η_i then

$$P_{i\ \mathrm{loss\ grid}}=(1-\eta_i)4\pi r_{\mathrm{grid}}^2\ n_{i\ \mathrm{grid}}\ v_{i\ \mathrm{grid}}\ E_{i\ \mathrm{grid}}. \quad (53)$$

Noting that $v_{i\ \mathrm{grid}}=\sqrt{2E_{i\ \mathrm{grid}}/m_i}$ and expressing the power in watts, energy in eV, and everything else in cgs units, one finds that

$$P_{i\ \mathrm{loss\ grid}}=2.79\cdot 10^{-12}(1-\eta_i) \times r_{\mathrm{grid}}^2\ n_{i\ \mathrm{grid}}\ E_{i\ \mathrm{grid}}^{3/2}/\sqrt{\mu_i}\ \mathrm{W}. \quad (54)$$

For $\mu_i=2$ and all other parameters as before, the ion losses will be about 60 times smaller than the corresponding electron losses calculated above; the reason is simply that the ions are moving much more slowly than electrons of the same energy. Unfortunately, the grid losses are still much greater than the fusion power:

$$\frac{P_{i\ \mathrm{loss\ grid}}}{P_{\mathrm{fus}}}\sim 40. \quad (55)$$

Because of the overwhelming power losses and cooling problems associated with grids, it would appear to be immensely preferable to use a different confinement technique for all but small-scale experiments.

G. Electron thermalization

Equation (41), the electron loss time for a cusp device with convergence-limited core density, may be compared with the electron–electron collision time,[36]

$$\tau_{\mathrm{ee}}=\frac{3\sqrt{3}\sqrt{m_e}T_e^{3/2}}{8\pi e^4 n_{e\ \mathrm{eff}}\ln\Lambda_{\mathrm{ee}}}=2.4\cdot 10^5\ \frac{T_e^{3/2}}{n_{e\ \mathrm{eff}}\ln\Lambda_{\mathrm{ee}}}, \quad (56)$$

where $n_{e\ \mathrm{eff}}$ is the square root of the mean square density experienced by an electron circulating through the plasma and T_e on the extreme right-hand side of the equation has been converted into eV. If the density profile may be approximated by Eq. (28) with $R\approx 100 r_c$, n_{eff} will be about an order of magnitude smaller than the core density.

It can be determined whether the electrons will be significantly thermalized by considering the ratio of the two times,

$$\frac{\tau_{\mathrm{ee}}}{\tau_{e\ \mathrm{loss\ cusp}}}=1.4\cdot 10^{13}\ \frac{T_{e0}^2 N k_{\mathrm{H}}^2 A(e\Phi_{\mathrm{well}})}{\ln\Lambda_{\mathrm{ee}} n_{e\ \mathrm{eff}} R^3 B^2}. \quad (57)$$

For typical values of the parameters involved (see, for example, Tables I and II), the ratio is found to be in the range

$$\frac{\tau_{\mathrm{ee}}}{\tau_{e\ \mathrm{loss\ cusp}}}\sim 10^{-6}-10^{-3}. \quad (58)$$

Thus it is readily apparent that electrons in the center of the IEC device will form an essentially Maxwellian distribution. As a result, one could not keep the very slow electrons that mediate ion–electron energy transfer[32] highly depleted in order to reduce the bremsstrahlung losses. (Also, one should not be tempted to increase the electron losses in order to facilitate the maintenance of nonthermal velocity distributions, since the electron losses are already intolerably large.)

H. Synchrotron radiation losses

In calculating the electron temperature and bremsstrahlung losses in Sec. II, the effects of synchrotron radiation were assumed to be negligibly small. This assumption will now be justified.

The power density of emitted synchrotron radiation is given in Ref. 36 as

$$\frac{P_{\mathrm{syn}}}{V}=\frac{4e^4B^2n_e}{3m_e^2c^3}\left(\frac{T_e}{m_ec^2}\right)\left[1+\frac{5}{2}\left(\frac{T_e}{m_ec^2}\right)\right]. \quad (59)$$

Evaluating the constants, defining V_{syn} to be the plasma volume which is under the influence of the magnetic field and emitting synchrotron radiation, letting f represent the fraction of the radiation which is actually lost (not reflected back into the plasma and reabsorbed there), and putting the mean electron energy $\langle E_e\rangle\equiv(3/2)T_e$ and the electron rest energy in eV, the synchrotron power becomes

$$P_{\mathrm{syn}}=4.14\cdot 10^{-28}B^2n_e\langle E_e\rangle\left[1+\frac{5}{3}\left(\frac{\langle E_e\rangle}{m_ec^2}\right)\right]fV_{\mathrm{syn}}\ \mathrm{W}. \quad (60)$$

In a diamagnetic IEC plasma, synchrotron radiation will only come from the outer layer of the plasma. Electron diamagnetism prevents the external magnetic field from penetrating more than a few electron gyroradii into the plasma.[10] The energy acquired from the ions via collisions in the center of the device will be substantially smaller than the well depth energy, so the electrons in the edge will have $\langle E_e\rangle\approx e\Phi_{\mathrm{well}}$. Using the fact that in this outer layer $B^2\approx 8\pi n_{\mathrm{edge}\ e}e\Phi_{\mathrm{well}}$ (with $e\Phi_{\mathrm{well}}$ in ergs) and defining the layer's thickness to be $k_{\mathrm{H}}\rho_e$, the synchrotron power is found to be

$$P_{\mathrm{syn}}\approx 1.67\cdot 10^{-38}f(e\Phi_{\mathrm{well}})^2 \times\left[1+\frac{5}{3}\left(\frac{e\Phi_{\mathrm{well}}}{m_ec^2}\right)\right]n_{\mathrm{edge}\ e}^2 4\pi R^2 k_{\mathrm{H}}\rho_e\ \mathrm{W}, \quad (61)$$

in which the well depth and electron rest energy are in eV.

The condition that $B^2\approx 8\pi n_{\mathrm{edge}\ e}e\Phi_{\mathrm{well}}$ allows the expression for the electron gyroradius from Eq. (38) to be rewritten in terms of the density:

$$\rho_e\approx 2.38\ \frac{\sqrt{2(e\Phi_{\mathrm{well}})_{\mathrm{eV}}}}{B}\ \mathrm{cm}=5.30\cdot 10^5\ \frac{1}{\sqrt{n_{\mathrm{edge}\ e}}}\ \mathrm{cm}. \quad (62)$$

With the aid of the relation $(n_{\mathrm{edge}\ e}/n_{ce})=(r_c/r_e)^2$, the ratio of the total synchrotron power to the total bremsstrahlung power may be estimated from the expression

$$\frac{P_{\mathrm{syn}}}{P_{\mathrm{brem}}}\sim 0.39\ \frac{fk_{\mathrm{H}}}{Z_{\mathrm{eff}}}\left(\frac{r_c}{r_e}\right)\left(\frac{R}{r_e}\right)^2\sqrt{\frac{e\Phi_{\mathrm{well}}}{T_{e0}}}\frac{(e\Phi_{\mathrm{well}})^{3/2}}{r_c\sqrt{n_{ce}}}, \quad (63)$$

TABLE I. IEC reactors utilizing deuteron-based fuels.

Parameter	D–T	D–^{3}He	D–D
Φ_{well}	60 kV	210 kV	300 kV
T_{i0}	20 keV	70 keV	100 keV
T_{e0}	18 keV	55 keV	75 keV
n_{ce}	$5\cdot10^{17}$ cm^{-3}	$5\cdot10^{17}$ cm^{-3}	$1\cdot10^{18}$ cm^{-3}
B	2.2 T	4.1 T	7.0 T
Fuel mixture	1:1	1:1	...
r_c	1.5 cm	2.5 cm	2.5 cm
$(\ln \Lambda)_{\text{average}}$	15	16	16
$\langle\sigma v\rangle_{\text{fus}}$ (10^{-16} cm^3/s)	4.31	1.04	0.495
E_{fus}	17.6 MeV	18.3 MeV	3.7 MeV
N_{cusps}	8	8	8
k_{H}	2	2	2
P_{fus}	4.3 GW (t)	2.2 GW (t)	3.8 GW (t)
$P_{\text{neutrons}}/P_{\text{fus}}$	0.80	0.01	0.36
$P_{\text{brem}}/P_{\text{fus}}$	0.008	0.24	0.47
$P_{e\text{ loss cusp}}/P_{\text{fus}}$ [from Eq. (46)]	0.11	1.32	1.29
$P_{e\text{ loss cusp}}/P_{\text{fus}}$ [from Eq. (A4)]	0.47	5.76	5.81
Ion fusion rate/loss rate ($\tau_{i\text{ loss}}/\tau_{\text{fus}}$)	D: $6\cdot10^{-3}$ T: $5\cdot10^{-3}$	D: $4\cdot10^{-3}$ ^{3}He: $2\cdot10^{-2}$	$6\cdot10^{-3}$
$\tau_{ii}/\tau_{\text{fus}}$	D: $1\cdot10^{-3}$ T: $1\cdot10^{-3}$	D: $2\cdot10^{-3}$ ^{3}He: $1\cdot10^{-4}$	$7\cdot10^{-4}$
$\tau_{ee}/\tau_{e\text{ loss cusp}}$	$1\cdot10^{-5}$	$2\cdot10^{-5}$	$9\cdot10^{-6}$

in which the temperatures and energies are in eV. (For this estimate the relativistic corrections to the synchrotron and bremsstrahlung losses were neglected, since they were of the same order of magnitude.)

Even at the rather extreme parameters of $e\Phi_{\text{well}}=10^6$ eV, $r_c=2$ cm, $n_{ce}=5\cdot10^{17}$ cm^{-3}, $e\Phi_{\text{well}}/T_{e0}\approx7$, $Z_{\text{eff}}=1$, $R/r_e=2$, and $r_c/r_e=1/50$, the ratio is only

$$\frac{P_{\text{syn}}}{P_{\text{Brem}}}\sim0.06fk_{\text{H}}. \tag{64}$$

Since $f\leqslant1$ and k_{H} should at most be 4 or 5, it is clear that bremsstrahlung, not synchrotron radiation, will be the dominant radiation loss mechanism. (Note that there is the luxury of further reducing the synchrotron losses by reflecting and reabsorbing most of the radiation in the plasma, thus making f much smaller than 1.)

This analysis of the synchrotron radiation losses has assumed that the plasma diamagnetically excludes the magnetic field except in a very thin sheath at the plasma surface. Even if such diamagnetic effects do not occur, the synchrotron losses will be limited by the fact that the vacuum magnetic field of the cusp system varies as $(r/R)^n$, where $n\geqslant3$.[19] Thus a strong magnetic field will still exist only near the plasma surface, although it might penetrate far enough so that the synchrotron losses become comparable to the bremsstrahlung losses. (In that event, however, the thicker magnetic layer at the edge of the plasma would result in larger effective cusp hole radii, so the electron cusp losses would become so severe that they would be a much more pressing concern than synchrotron losses.)

IV. RESULTS

Tables I and II present parameters and results for IEC reactors using various fuels. The reactors are assumed to use a magnetic cusp confinement system but not to utilize the ICC effect to enhance the core densities beyond those of normal convergence-limited flow. In each case parameters such as the fuel mixture and reactor size have been chosen so that the reactor performance is approximately optimized while keeping the reactor's specifications (for instance the total power output) within reasonable limits.

For each fuel the ion temperature in the tables has generally been chosen to approximately minimize $P_{\text{brem}}/P_{\text{fus}}$ rather than the relative power loss due to electrons leaking out of the magnetic cusps. The reason for this choice is that the bremsstrahlung loss is a fairly universal problem which one must simply live with, whereas it is hoped that the (extremely large) electron cusp losses can be controlled by somehow improving the confinement system. Values of both the bremsstrahlung and the cusp losses at other ion temperatures are graphed in Figs. 1–6 for each fuel type.

The ions are assumed to have an initially monoenergetic distribution at an energy $E_{i0}=Z_{i1}e\Phi_{\text{well}}/2$, where Z_{i1} is the charge of the lowest Z ion species present. (Of course, to produce such an initial distribution one would have to surmount the difficulties of very accurately injecting ions that deeply into the well, but for the present calculations those technical problems are neglected.) Since $\langle E_{i1}\rangle\approx\langle E_{i2}\rangle$, as shown in Sec. II B, any higher Z_i ion species which are present will have to be injected even more deeply into the well. The ions will begin to evolve toward a Maxwellian distribution with $T_{i0}=(2/3)E_{i0}$ on a time scale of τ_{ii}. As is

TABLE II. IEC reactors utilizing advanced aneutronic fuels.

Parameter	p–^{11}B	p–^{6}Li	^{3}He–^{3}He
Φ_{well}	900 kV	1.5 MV	1.5 MV
T_{io}	300 keV	500 keV	1.0 MeV
T_{eo}	138 keV	204 keV	278 keV
n_{ce}	$5\cdot10^{17}$ cm^{-3}	$1\cdot10^{18}$ cm^{-3}	$5\cdot10^{17}$ cm^{-3}
B	8.5 T	15.5 T	11.0 T
Fuel mixture	5:1 p:^{11}B	3:1 p:^{6}Li	...
r_c	3 cm	3 cm	3 cm
$(\ln\Lambda)_{\text{average}}$	16	16	17
$\langle\sigma v\rangle$ (10^{-16} cm^3/s)	2.39	1.1	1.25
E_{fus}	8.7 MeV	4.0 MeV	12.9 MeV
N_{cusps}	8	8	8
k_{H}	2	2	2
P_{fus}	1.9 GW (t)	2.7 GW (t)	3.7 GW (t)
$P_{\text{brem}}/P_{\text{fus}}$	1.74	5.36	1.42
$P_{e\text{ loss cusp}}/P_{\text{fus}}$ [from Eq. (46)]	10.5	15.1	12.9
$P_{e\text{ loss cusp}}/P_{\text{fus}}$ [from Eq. (A4)]	61.1	92.5	67.5
Ion fusion rate/loss rate ($\tau_{i\text{ loss}}/\tau_{\text{fus}}$)	p: $3\cdot10^{-2}$ ^{11}B: 330	p: $5\cdot10^{-2}$ ^{6}Li: 3	$4\cdot10^{-2}$
$\tau_{ii}/\tau_{\text{fus}}$	p: $5\cdot10^{-3}$ ^{11}B: $7\cdot10^{-4}$	p: $8\cdot10^{-3}$ ^{6}Li: $2\cdot10^{-3}$	$4\cdot10^{-3}$
$\tau_{ee}/\tau_{e\text{ loss cusp}}$	$7\cdot10^{-5}$	$4\cdot10^{-5}$	$3\cdot10^{-4}$

indicated for each fuel species in the tables, the thermalization of a test ion typically occurs two to three orders of magnitude more rapidly than the fusion of that test ion. Thus the ion distributions in the center of the device will be essentially Maxwellian, except that the high-energy tail will be truncated at the well depth.

Neglecting this truncation, the appropriate reactivity to use in calculating the fusion time τ_{fus} for comparison with the thermalization time is the beam-Maxwellian quantity, since the test ions just entering the system have only begun to thermalize. However, under these conditions where the beam energy is 3/2 of the Maxwellian temperature, the reactivity can be well approximated to within a few percent by the Maxwellian-averaged reactivity; hence the Maxwellian-averaged $\langle\sigma v\rangle$ values are used for simplicity. Cross-section data is drawn from Refs. 48–50.

The tables also give the fraction of ions that fuse before they can be lost over the top of the potential well, which is the ratio of the fusion rate to the loss rate, or $\tau_{i\text{ loss}}/\tau_{\text{fus}}$. Since the lower Z_i ion species in each system has a mean energy of half the well depth, its members will escape much more rapidly than they will fuse. Ions with higher values of Z_i will be confined better, but this result assumes that the ions can be injected correspondingly more deeply into the well so that they will have the same mean energy as the lower Z_i ion species. Confinement of all types of ions could

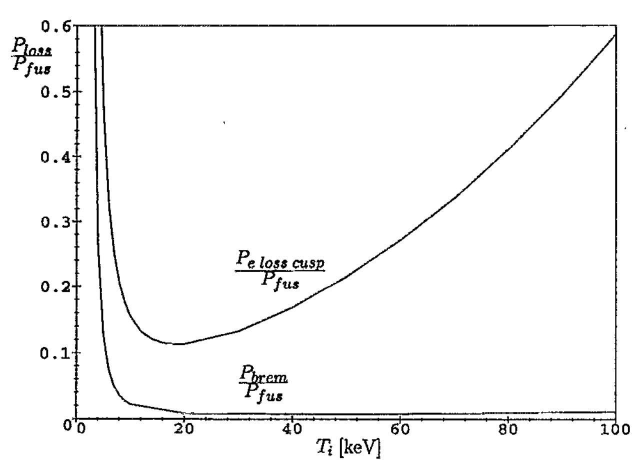

FIG. 1. Ratios of electron cusp losses and bremsstrahlung losses to fusion power for D–T. (System parameters given in Table I.)

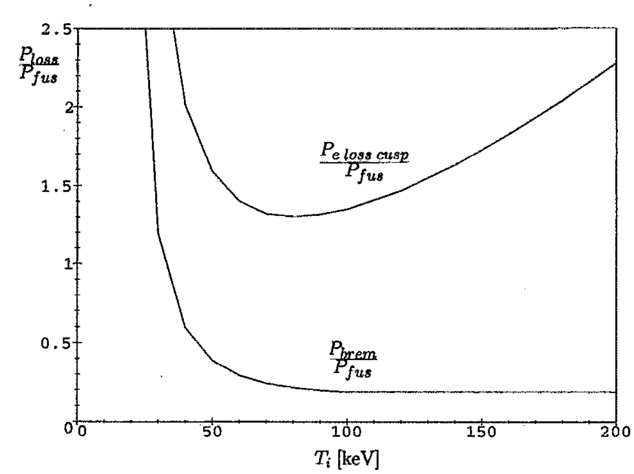

FIG. 2. Ratios of electron cusp losses and bremsstrahlung losses to fusion power for D–^{3}He. (System parameters given in Table I.)

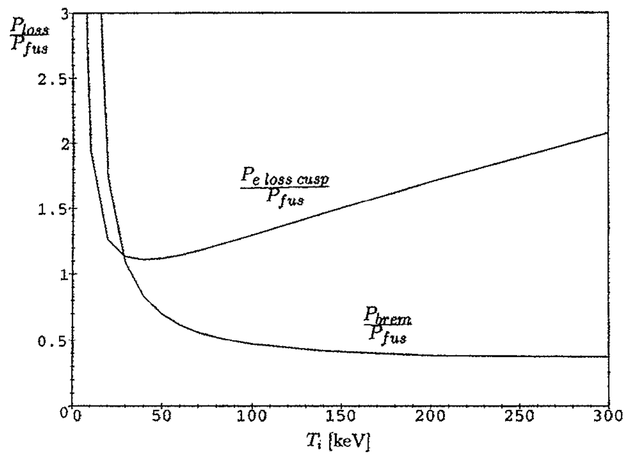

FIG. 3. Ratios of electron cusp losses and bremsstrahlung losses to fusion power for D–D. (System parameters given in Table I.)

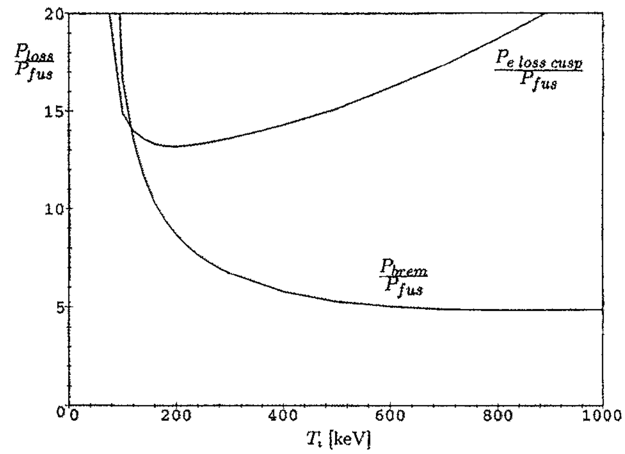

FIG. 5. Ratios of electron cusp losses and bremsstrahlung losses to fusion power for p–^{6}Li. (System parameters given in Table II.)

be improved by even deeper injection into the well, but deep injection poses tremendous practical problems.

Because both of the ion species are nearly Maxwellian with equal temperatures, the averaged fusion cross sections will be considerably smaller than they would be if the ions could be maintained so that one species was at very low energy and the other was monoenergetic at the resonance peak energy. In fact, since the high-energy ion tail escapes from the system, the average reactivities of fuels in an IEC device will be somewhat less than those in a fusion reactor which can confine the hot ion tail of the Maxwellian. For simplicity, Maxwellian-averaged reactivities have been used in calculating the fusion power, but it must be remembered that the true reactivity will be smaller and so the power loss fractions will be somewhat larger than shown.

Figures 1–6 plot the ratios of bremsstrahlung and electron cusp losses to the fusion power for a wide range of fuel types and ion temperatures. While the bremsstrahlung power loss is quite small for D–T, tolerable for D–^{3}He, and perhaps tolerable for a pure D system, it is prohibitively large for all other fuels, since the high ion energies in the center of the device lead to high electron temperatures there as well. Because of the truncated Maxwellian ion distributions, the bremsstrahlung/fusion ratios will be roughly equal to or perhaps even worse than those of other reactors burning the same fuels. (No Maxwellian-averaged reactivity data could be found for pure ^{3}He plasmas with ion temperatures above 1 MeV, but the losses are clearly leveling off at prohibitively large values. Also, at $T_i=1$ MeV one is already putting 3 MeV of energy into each ion pair in order to get 12.9 MeV out of each fusion event, so it would not be terribly advantageous energetically to have to increase the energy input to the ions much more by raising the temperature further, especially considering the number of large energy loss mechanisms which are present.) It has been assumed that all fusion products will escape over the top of the potential well, so the fusion power calculated here does not account for any extra energy which could be derived from burning the fusion products. Even if fusion products were burned, however, only the performance of D–D would be improved to a useful degree

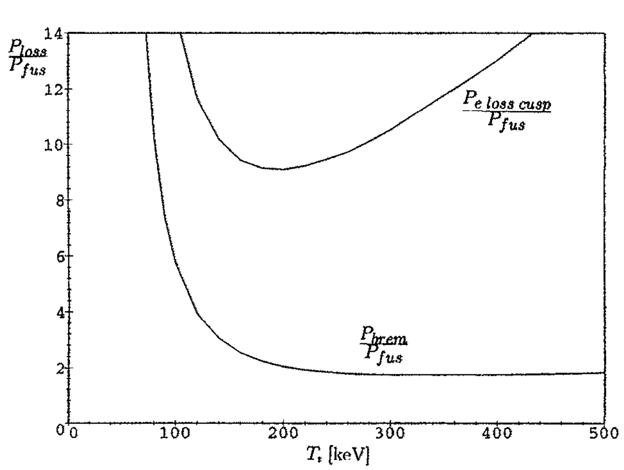

FIG. 4. Ratios of electron cusp losses and bremsstrahlung losses to fusion power for p–^{11}B. (System parameters given in Table II.)

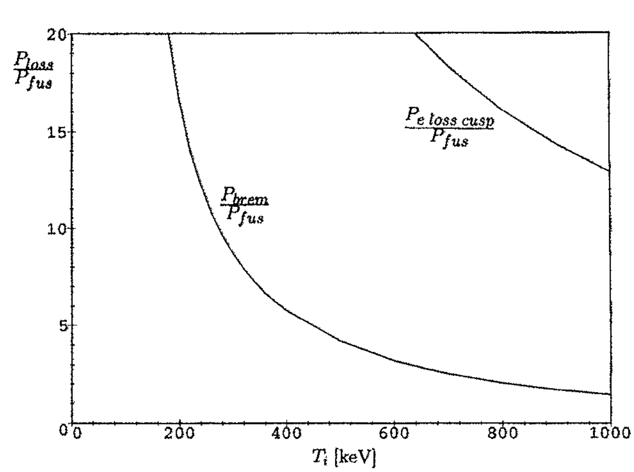

FIG. 6. Ratios of electron cusp losses and bremsstrahlung losses to fusion power for ^{3}He–^{3}He. (System parameters given in Table II.)

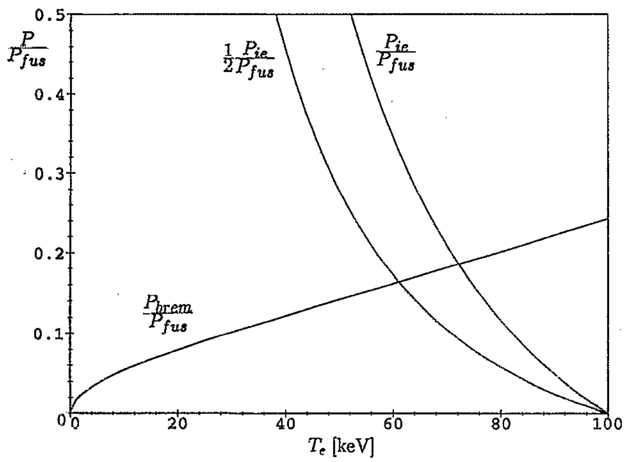

FIG. 7. Ion–electron heat transfer and bremsstrahlung for D–^{3}He with various electron temperatures. (T_i=100 keV.)

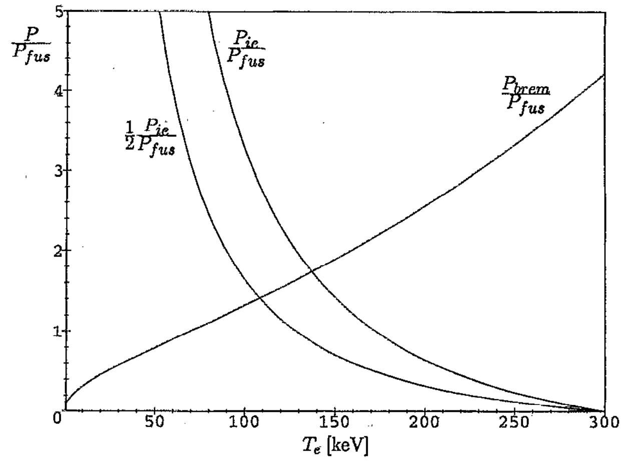

FIG. 9. Ion–electron heat transfer and bremsstrahlung for p–^{11}B with various electron temperatures. (System parameters given in Table II.)

(p–^{6}Li would still be unable to break even against bremsstrahlung, despite the burnup of its product ^{3}He with ^{6}Li or exogeneous D), whereas several of the fuel mixtures would experience undesirable increases in neutron production.

Figures 7–11 graphically show the operating points for various fuels, based on the intersection of the ion–electron energy transfer curve and the bremsstrahlung radiation curve. One should note that the expression used here for the ion–electron heat transfer incorporates all of the necessary correction factors due to relativistic effects, ion-induced partial depletion of slow electrons, the dependence of the Coulomb log on the electron temperature, and the possibility that the mean ion speed may become comparable to the mean electron speed. As shown in these figures, even if one could find a method for cutting the ion–electron heat transfer rate in half, the operating point of the reactor would not change enough to lower the radiation losses sufficiently for any of the more advanced fuels to become viable. Furthermore, if one attempted to cool the electrons actively, one would have to extract considerably more power from them than one would obtain in the form of fusion power (as revealed by the behavior of the $P_{ie}/P_{\rm fus}$ curves for low electron temperatures). Even if there were a mechanism for recirculating this vast amount of power from the electrons back to the ions, it almost certainly would not be able to do so with sufficiently small losses.

These bremsstrahlung loss calculations are especially important because they apply not just to IEC devices but also to virtually any other fusion system in which the regions of appreciable density are approximately isotropic, quasineutral, and optically thin to bremsstrahlung, and in which the ions are maintained at a fixed mean energy ($3T_{i0}/2$). While it is true that the bremsstrahlung losses could be reduced by cooling the electrons via other loss mechanisms (e.g. synchrotron radiation or the loss of high-energy electrons), such techniques would not be useful from the point of view of net power losses for the reactor system. Also, if the electrons are heated by sources other than simple Coulomb friction with the fuel ions (for example, external RF heating or friction with the fusion products), the electron temperature and bremsstrahlung losses will be even higher than shown here.

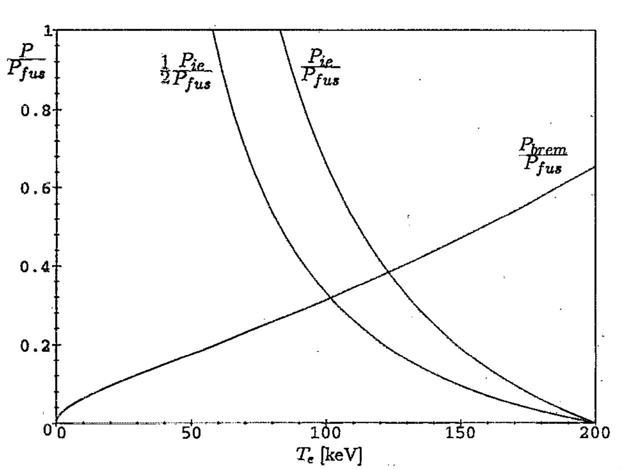

FIG. 8. Ion–electron heat transfer and bremsstrahlung for D–D with various electron temperatures. (T_i=200 keV.)

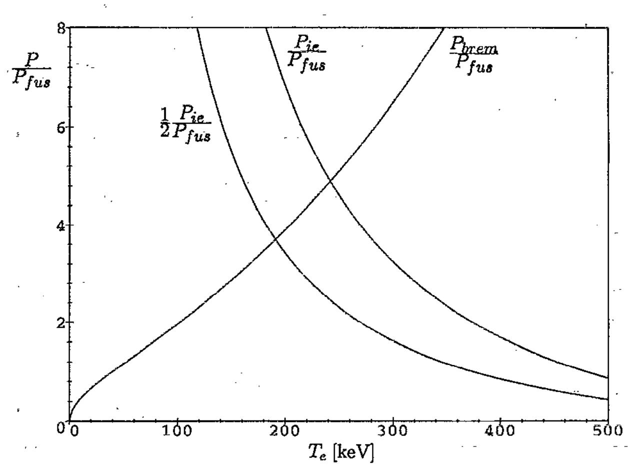

FIG. 10. Ion–electron heat transfer and bremsstrahlung for p–^{6}Li with various electron temperatures. (T_i=700 keV.)

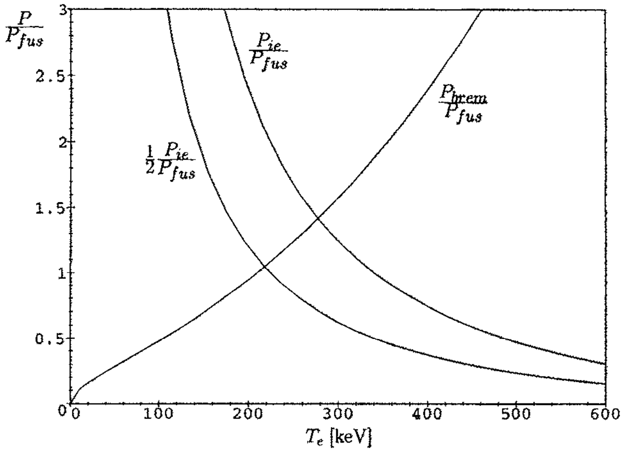

FIG. 11. Ion–electron heat transfer and bremsstrahlung for $^3He-^3He$ with various electron temperatures. (System parameters given in Table II.)

It must be emphasized that the electron cusp losses shown in Figs. 1–6 are based on the most optimistic calculations that can be reasonably justified. Equation (46) was used instead of the more pessimistic Eq. (A4), which predicts losses approximately four times larger (see the tables and the Appendix for a comparison of the results of the two equations). The cusp holes were assumed to have a radius of only twice the electron gyroradius ρ_e, when, in fact, they may be several times larger (according to work by Grad[44] and Grossman[45] the hole radius could be as large as five times the electron gyroradius) or even scale like $\sqrt{\rho_e \rho_i}$ when ion effects are included.[47] It was also optimistically assumed that very high core densities could be achieved and that Ohmic power losses in the field coils could be neglected. Furthermore, it was assumed that the magnetic field configuration was a perfect octahedron with losses only at the $N=8$ point cusps, when in reality there may also be line cusps, substantial electron transport across magnetic field lines, or other sources of electron losses, and to avoid perturbing the desired spherical symmetry of the system seen by the ions it would in fact probably be necessary to go to a system with $N=20$ cusps or more.[51,52] [In obtaining the results shown, assumptions made about the potential well shape and density profiles were that $A=1.5$, $R=2r_e$, and $r_e=50r_c$. The derivation of electron losses culminating in Eq. (A4) did not need to make assumptions about such parameters, but its results proved even more pessimistic, as already noted.]

Even by making all of these optimistic assumptions and adding direct converters to extract energy from escaping electrons with 50%–60% efficiency, the electron losses are found to be intolerably large for fuels other than D–T. Moreover, since the fusion power, as given in the tables, is already several gigawatts, it is not practical to reduce the relative severity of the cusp losses by increasing the fusion power.

The strength of the cusp magnetic field, as given in the tables, is calculated assuming that $\beta=1$ at the outer plasma surface,[10,46] or $B^2=8\pi n_{\text{edge }e} e\Phi_{\text{well}}$ with $e\Phi_{\text{well}}$ in ergs, where $n_{\text{edge }e}=n_{ce}(r_c/r_e)^2$. (It would be possible to reduce the electron cusp losses if the outer layer of the diamagnetic plasma could be maintained in equilibrium with $\beta<1$, so that higher magnetic field strengths could be used. However, the behavior of the outer sheath of the diamagnetic plasma is poorly understood, and the plasma might simply adjust itself to keep $\beta=1$, as assumed in Ref. 10. In any event, the magnetic field strengths indicated in the tables are already quite large, so it would be rather difficult to increase them much more.)

Under actual conditions, the electron cusp losses will almost certainly be far more severe than the extremely optimistic values calculated here, thus preventing even D–T from achieving a positive power balance.

V. CONCLUSIONS

The suitability of various implementations of IEC systems for use as D–T, D–D, D–^{3}He, p–^{11}B, p–^{6}Li, and ^{3}He–^{3}He reactors has been examined. It has been shown that while an IEC reactor would have the advantages of high power densities and relatively simple engineering design when compared with other fusion schemes, it suffers from several flaws. These problems include ion thermalization and upscattering losses, bremsstrahlung radiation, and electron cusp losses. Other issues, such as the potential usefulness of the ICC effect, have also been examined.

A. Ion thermalization and upscattering losses

The problem of ion thermalization and upscattering has been examined in detail. Since the local thermalization rate due to Coulomb collisions and the local fusion rate both have the same dependence on density, integrating over the spatial variations of density in an IEC device leaves the ratio of the thermalization time to the fusion time unaffected. One finds the ratio to be the same as in other fusion reactor designs, namely that an ion thermalizes about two to three orders of magnitude faster than it fuses. Therefore, it appears that the ion velocity distribution in the broad flat bottom of the potential well will look essentially like a Maxwellian truncated at the well depth, rather than the desired monoenergetic distribution. Furthermore, energy is transferred between the two ion species on a time scale roughly comparable to the thermalization time of each individual species, so that it would not appear possible to maintain the two ion species at significantly different energies or temperatures in order to take better advantage of the resonance peaks in the reaction cross sections.

If the initial ion energy is not much smaller than the potential well depth, say half of the well depth (it would be quite difficult to inject fresh ions even that far into the well and with great accuracy), then ions will not have to be scattered terribly far out into the tail to be lost. They can cover this comparatively small distance in velocity space in just a few ion–ion collision times. Thus not only will the ions thermalize far more rapidly than they will fuse, but they will also escape the well much more rapidly than they fuse (except for ions like ^{11}B which see a much deeper well). It would be possible to reduce the ion losses by employing deeper injection and correspondingly stronger confining potentials, but deep injection into the well would be a very difficult problem, and larger well depths would lead to even greater power losses due to escaping electrons. The rapid loss of ions over the top of the potential well would at the very least make IEC systems extremely impractical as reactors (due to the need to continually collect and reinject the vast number of ions), and may even prohibit these systems from producing net power, if the ions have acquired a substantial amount of perpendicular energy before escaping.

It has recently been suggested[37] that the problems with rapid collisional relaxation of the ion and/or electron distributions might be overcome if collisions within the dense central region of the plasma behaved in a highly anisotropic one-dimensional fashion rather than in a three-dimensional isotropic fashion, so that the collisional relaxation effects would happen on a much longer time scale. This behavior would require that the space-charge repulsion from a "hard core" of extremely low-energy particles trapped at the very center of the device would reflect incoming particles of the same species before they entered the isotropic region of the core.

Unfortunately, this method appears to be extremely unlikely to succeed. For it to work as desired, virtually all of the particles must bounce straight back from the hard core so that they only encounter other particles in head-on collisions. Even if a repulsive core is established, a more realistic outcome would be that a considerable number of particles converging toward the center would be somewhat deflected to the side by the core and then "sideswiped" by other incoming particles, leading to collisional behavior that is approximately isotropic. Another difficulty is that there will be a high turnover rate (due to the fast collisional rates) of the very low energy particles which constitute the hard core, as they exchange places with some of the higher-energy particles in the system. Thus most of the particles in the device will get to circulate into the isotropic core and undergo collisions there. (In other words, the core would not have a well-defined and impenetrably hard boundary.) Finally, even if the dense central region of an IEC system can be made to behave in a strongly anisotropic fashion, this behavior would be highly detrimental when instabilities are considered.[31] Nonetheless, in the future it would be interesting to use particle simulations to see if an anisotropic hard core can be maintained to any significant degree, and if this would have an appreciable effect on the collisional processes.

B. Bremsstrahlung

The radiated bremsstrahlung power will, of course, depend on the rate of the energy transfer between ions and electrons which occurs primarily near the dense core. Just as the ion–ion collision time is much shorter than the fusion time, it can also be easily shown that the electron–electron collision time is many orders of magnitude shorter than the electron loss time in the magnetic cusp confinement system. Therefore, both the ions and electrons will have roughly Maxwellian distributions in the device center; the temperature of the electrons relative to that of the ions must then be determined.

For a given ion temperature, the equilibrium value of the electron temperature is obtained by equating the standard Spitzer-type ion–electron heat transfer rate (accounting for modifications due to ion-induced depletion of slow electrons, relativistic electron effects, and the temperature dependence of the Coulomb log) with the bremsstrahlung cooling rate of the electrons; this derivation yields electron temperatures which are large enough to make bremsstrahlung losses prohibitively large at the very high ion energies necessary for p–^{11}B, p–^{6}Li, ^{3}He–^{3}He, and perhaps even pure D–D reactions. It was shown that even if some method could be found to reduce the ion–electron heat transfer by half, the bremsstrahlung losses would not be lowered sufficiently to be of any use.

These calculations of bremsstrahlung losses are especially important because they apply to a wide variety of other fusion reactor schemes as well.

C. Electron cusp losses

The power loss caused by electrons escaping through the magnetic cusps was found to be roughly proportional to $\Phi_{\mathrm{well}}^{3/2}$, where Φ_{well} is the well depth. Even by using the most optimistic assumptions which could be justified (an octahedral cusp system in which the radius of each cusp hole is only twice the electron gyroradius), the cusp losses were intolerably large for all fuels except D–T, which could operate with a much shallower well depth than the other fuels. However, quite likely D–T will also prove to have prohibitively large cusp losses when one accounts for practical limitations on how small the cusp holes can actually be made and how many cusps are required to adequately approximate a spherically symmetric system in order to avoid destroying proper ion convergence.

If electrostatic grids are used instead of magnetic cusps, the electron losses should be orders of magnitude worse (thus preventing break-even with any fuel), large numbers of ions would also be lost by collisions with the grids, and the severe problem of grid heating would also arise. While grids are convenient for small-scale experiments, they do not appear to be desirable in actual IEC reactors.

The only apparent route to possibly improving the electron confinement would seem to be the use of Penning traps or higher-order multipole systems which have been proposed,[14–16] although it is not yet known if the electron losses from these systems will actually be tolerably small, especially when large numbers of ions are also present. Even if such systems are able to lower electron losses to reasonable levels, it is far from certain that these concepts could be scaled up to large (from MW to GW range) power-producing reactors, and even then they would still be subject to the thermalization problems, ion upscattering losses, and bremsstrahlung radiation losses which have already been described.

D. Acoustic-wave compression of the core

Although the use of acoustic standing waves to increase the core density and/or alter the density profile has been proposed in both Refs. 22 and 23, it appears that such a phenomenon could do little to improve the fundamental problems noted above (and it may even have a detrimental impact). For example, the ratios of ion thermalization and upscattering times to the fusion time are independent of both the core density and the spatial profile of the density in the reactor, and so they would remain unaffected by the so-called ICC effect. Likewise the ratio of bremsstrahlung power to fusion power would also remain the same.

One might think that using the ICC effect to increase the core density relative to the edge density would improve the ratio of cusp losses to fusion power, since the cusp losses occur at the edge and fusion occurs in or near the core. However, the constraint that $\beta=1$ for equilibrium at the outer plasma boundary effectively nullifies this possibility. If the core density is held constant and the ICC effect is invoked to decrease the edge density by a factor of ξ (in an attempt to reduce the losses), the equilibrium magnetic field B in the edge will decrease by a factor of $\sqrt{\xi}$, in accordance with the $\beta=1$ constraint. Since the cusp hole radius is inversely proportional to the magnetic field, the area of the cusp holes will increase by a factor of ξ, thereby completely negating any beneficial effect one might have hoped to gain from the decreased edge density. (See the Appendix for more details.)

The only critical parameter is the core density, which may be created via the ICC effect or simply by unaided ion flow convergence at the center of the device. Obviously the primary effect of altering the core density will be to change the fusion power density and total fusion power, and the calculations summarized in Tables I and II already use reactor designs with fusion power levels as high as can reasonably be tolerated.

Even if it were quite desirable to employ the ICC effect, it is far from certain that the acoustic waves will work as expected to compress the core. If the ICC effect does indeed occur, it is highly questionable whether it can achieve the necessary many-fold compression without simultaneously degrading the central ion convergence and defeating the purpose of its use.

E. Other potential problems

There are several other issues which were not examined in this paper but which would need to be carefully considered in future IEC work. These areas include the following.

Limitations on core convergence: It is necessary to determine the limitations on maximum core density and more closely scrutinize the rate of core spreading due to angular momentum buildup. Collisions in and near the dense core will tend to increase the angular momentum of ions and degrade the central focus on a time scale comparable to the ion collision time, which was shown to be much faster than the fusion time. Only by invoking collisional effects in the edge has it been argued that core convergence can be maintained,[19] and as was shown in this analysis, collisional effects in the edge are far smaller than those in the center of the plasma. Thus the arguments presented in Ref. 19 should be reexamined, especially taking into account the fact that the ion distributions will be radially Maxwellian, not monoenergetic.[29]

Counterstreaming instabilities: One would also have to determine whether counterstreaming instabilities will saturate at a tolerable level or if they can somehow be avoided; such an investigation must take into account the fundamental nonlinear, nonlocal nature of the problem.[21]

Anisotropic instabilities: Instabilities arising from the inherent anisotropy of the system (such as the Weibel electromagnetic instability[30,31]) must also be examined. While these instabilities should not directly affect the core, which is approximately isotropic, they may have a great negative impact on the velocity distributions in the mantle and edge of the plasma and thereby prevent large core densities or otherwise disrupt the desired operating characteristics of the system.

Lifetime of the potential well: Another question that should be examined is whether part or all of the potential well will eventually fill in due to background neutrals or other possible neutralizing effects. If such effects do occur, one should determine whether their time scale will seriously limit the time for which the IEC device can operate before it must flush out its contents.

Deviations from spherical symmetry in the system: As has been pointed out,[51,52] deviations of the magnetic field from spherical symmetry may have extremely deleterious effects on the ion orbits and core convergence. Whereas the present study has quite optimistically assumed that an octahedral cusp system would be a satisfactory shape, it would quite likely be necessary to go to a much higher-order polyhedral magnetic field shape to permit proper potential well formation and core convergence. This would increase not only the complexity of the system but also the power loss due to escaping electrons.

Technological issues. There are serious technological problems which must be explored, such as finding suitable techniques for accurately fueling deep inside the well and designing direct converters which can cover most of the 4π steradians around spherical IEC devices (or even function at all, given the serious difficulties with direct converters described in Ref. 28).

F. Outlook for the future

While this analysis of IEC devices has employed extremely optimistic assumptions about the particle convergence and electron confinement in such systems (in accordance with the assumptions about those areas as described in Ref. 1), it has found numerous fatal flaws in the fundamental IEC concept. Although the calculations presented in this paper have admittedly been fairly simple analytical treatments of the problems, the results of the calculations indicate that ion thermalization, electron thermalization, ion losses, electron losses, and ion–electron energy transfer (leading to bremsstrahlung losses) will happen *orders of magnitude* too rapidly for IEC devices to yield the performance that has been claimed by proponents of the concept. One might hope to gain a factor of 2 in a particular area due to some subtle effect which would be revealed by experiments or more sophisticated calculations, but surely not multiple orders of magnitude in all of these areas of difficulty. And although it might be possible to improve the device performance substantially by adding several additional systems to reduce the particle losses, thermalization, etc., IEC devices would then lose their proclaimed engineering simplicity and become just another extremely technologically complex fusion approach.

It is hoped that discussion of these problems with IEC will lead to the discovery of more radical and fundamental methods for circumventing them without losing the attractive engineering simplicity of IEC devices. Certainly it would be very welcome to have a fusion approach which could be thoroughly proven in fairly small-scale, simple experiments and yet could scale up to an economically attractive reactor capable of using D–^{3}He and perhaps even more advanced fuels.

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APPENDIX: ALTERNATE DERIVATION OF CUSP LOSSES

One may derive the cusp electron losses in a different fashion than was done in Sec. III D; the advantage of this alternate derivation is that it does not depend on parameters associated with the exact shape of the potential well and particle density profiles.

Assuming a spherically symmetric spatial distribution of electrons with essentially radial velocities, the number of electrons that escape each second will be $\frac{1}{2} n_{\text{edge}\ e} v_e$ times the total area of the cusp holes. The factor of 1/2 is included because only half of the electrons are traveling outward. If each escaping electron carries away an amount of energy E_{loss} and there are N cusps (thus $N\pi r_{\text{H}}^2$ is the total hole area), then the power loss due to escaping electrons will be

$$P_{e\ \text{loss cusp}} = \frac{1}{2} N\pi r_{\text{H}}^2 n_{\text{edge}\ e} v_e E_{\text{loss}}. \tag{A1}$$

Noting that $E_{\text{loss}} = E_e \approx e\Phi_{\text{well}}$, the well depth energy, and that $v_e = \sqrt{2E_e/m_e}$, and then expressing the power in watts, energy in eV, and everything else in cgs units, one obtains

$$P_{e\ \text{loss cusp}} \approx 1.69 \cdot 10^{-10} \frac{N k_{\text{H}}^2 n_{\text{edge},\ e} (e\Phi_{\text{well}})^{5/2}}{B^2}\ \text{W}. \tag{A2}$$

If there is no ICC enhancement of the core density, then the fraction of power which is lost because of escaping electrons is

$$\frac{P_{e\ \text{loss cusp}}}{P_{\text{fus}}} \approx 6.30 \cdot 10^7 N k_{\text{H}}^2 \frac{(x+Z_2)^2}{x} \times \frac{1}{\langle \sigma v \rangle E_{\text{fus}}} \frac{n_{\text{edge}\ e} (e\Phi_{\text{well}})^{5/2}}{B^2 n_{ce}^2 r_c^3}. \tag{A3}$$

If the ICC effect is used to enhance the core density significantly, the vast majority of the fusion power will only come from the core, so the above expression should be multiplied by four.

Now one makes the assumption that the edge of the plasma has $\beta = 1$,[10,46] or $B^2 \approx 8\pi n_{\text{edge}\ e} e\Phi_{\text{well}}$, for $e\Phi_{\text{well}}$ in ergs. Putting the energy in eV and substituting into Eq. (A3), the ratio of cusp loss power to fusion power becomes

$$\frac{P_{e\ \text{loss cusp}}}{P_{\text{fus}}} \approx 1.56 \cdot 10^{18} N k_{\text{H}}^2 \frac{(x+Z_2)^2}{x} \times \frac{1}{\langle \sigma v \rangle E_{\text{fus,eV}}} \frac{(e\Phi_{\text{well}})_{\text{eV}}^{3/2}}{n_{ce}^2 r_c^3}. \tag{A4}$$

It should be noted that this answer is about 4 times larger than the result of the electron loss derivation that was presented in Sec. III D. If this more pessimistic formula is used, the cusp losses for D–T become high enough to consume almost half of the fusion power, even if one still makes the optimistic assumptions that electrons are only lost at 8 point cusps (a perfect octahedral point cusp confinement system) and that the radius of the cusp holes is only twice the electron gyroradius. Losses for other fuels, of course, would become even more prohibitive than the values that have already been shown.

One might think that using the ICC effect to increase the core density relative to the edge density would improve the ratio of cusp losses to fusion power, since the cusp losses occur at the edge and fusion occurs in or near the core. Yet, as indicated in Eq. (A4), the constraint that $\beta = 1$ at the outer plasma boundary removed the dependence of the power loss fraction on the edge density, and thus on the density profile. It appears that using acoustic waves to alter the density profile of the device will create no significant improvement in reactor performance, provided that the waves only act to alter the density profile. The only critical parameter is the core density, which may be created via the ICC effect or simply by unaided ion flow convergence at the center of the device.

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